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Equilibration length of electrons in spin-polarized edge channels

G. Müller, D. Weiss, A.V. Khaetskii,* K. von Klitzing, and S. Koch
Max-Planck-Institut für Festkörperforschung, Heisenbergstrasse 1, D-7000 Stuttgart 80, Federal Republic of Germany

H. Nickel, W. Schlapp, and R. Lösch
Forschungsinstitut der Deutschen Bundespost, D-6100 Darmstadt, Federal Republic of Germany

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We investigate experimentally the "mean free path" of electrons injected selectively into the outer spin-polarized edge channel of the lowest Landau level. A simple model allows us to determine the characteristic length between two spin-flip scattering events directly from the measured resistance plateaus. Typical spin-flip equilibration lengths between 100 μm and 1 mm are found. The size of the observed equilibration lengths is interpreted in terms of the spin-orbit interaction, which also leads to a local g-factor enhancement at the edge.

The concept of one-dimensional current-carrying edge channels has been sucessfully applied to explain the magnetotransport properties of a two-dimensional electron gas (2DEG) in the quantum Hall regime.[1,2] Within this approach, the edges of a device play a significant role since they energetically "bend up" the Landau levels[3] and their intersection with the Fermi energy forms the one-dimensional channels which govern the transport properties. A new transport regime, characterized by a lack of local equilibrium over macroscopic distances (adiabatic transport) for selectively populated edge channels has attracted recent interest.[4–7] The lack of local equilibrium is attributed to the suppression of interedge channel scattering.[8] In the case of spin-degenerate edge channels, this is thought to be a consequence of their large spatial separation which drastically reduces the overlap of their wave functions. As far as spin-split edge channels are concerned, however, the situation is expected to be different, both because the spin gap is much smaller than the Landau gap and because the interedge channel scattering process has to be accompanied by a spin-flip process.

In this paper, the scattering between the two spin-polarized edge channels of the lowest Landau level is investigated. We deduce equilibration lengths (distance between interchannel scattering events) directly from four-point resistance measurements. We interpret our results as a consequence of spin-orbit interaction and estimate theoretically typical equilibration lengths.

Figure 1(a) shows the layout of our devices patterned from high-mobility MBE (molecular-beam epitaxy) grown GaAs-$Al_xGa_{1-x}As$ heterostructures. The heterojunction material, discussed here, was grown in the [100] direction on a semi-insulating substrate and consists of a 2.4-μm undoped GaAs buffer layer, 340-Å undoped $Al_{0.4}Ga_{0.6}As$, 340-Å silicon-doped $Al_{0.4}Ga_{0.6}As$, and a GaAs cap layer of 200 Å. At helium temperature, the devices have a carrier density of 1.9×10^{11} cm^{-2} and a mobility of 1.6×10^6 cm^2/V s corresponding to an electron mean free path of 12 μm at zero field. On top of a conventional Hall bar geometry, two Schottky gates [Fig. 1(a)] are evaporated across the mesa. By applying a negative gate bias two potential barriers are formed within the 2DEG, allowing the transmission and reflection probabilities of the edge channels to be tuned by the gate voltage. We use these barriers to selectively populate the edge channels. Figure 1(a) shows two edge channels sketched for the case of a bulk filling factor $b = 2$, adjusted by the magnetic field B, and a filling factor of $g = 1$ underneath both Schottky gates. The inner-edge channel, circulating between the gates, can couple to the

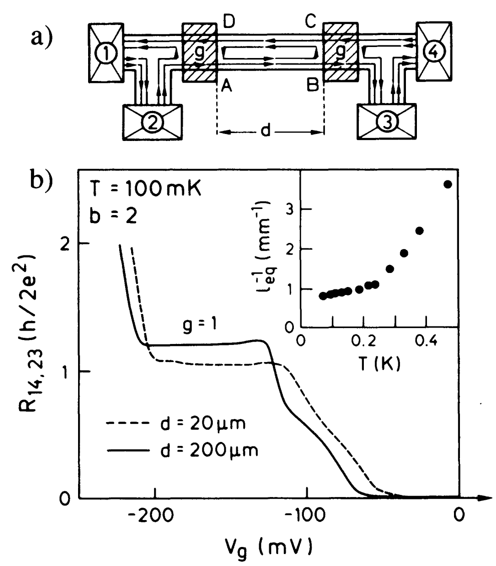

FIG. 1. (a) Schematic layout of our devices. The edge channels for $b = 2$ and $g = 1$ are sketched. (b) Gate voltage dependence of $R_{14,23}$ for two gate finger spacings with $b = 2$ at $T = 100$ mK for $I = 10$ nA. The voltage applied to both gates [shaded areas in (a)] is the same. The inset shows the temperature dependence of the inverse equilibration length for $d = 200$ μm. The data in the inset are taken after thermal cycling; the equilibration lengths are slightly higher compared to the previous cycle.

transmitted one *only* via spin-flip scattering processes.

In the experiments, the longitudinal four-point resistance $R_{14,23}$ is measured. The subscripts 14,23 indicate that the current is applied between the contacts 1 and 4 and the corresponding voltage drop is measured between the contacts 2 and 3. Within the Landauer-Büttiker formalism[9] this resistance can be expressed by the transmission T, which is the sum over the transmission probabilities of the edge channels across the barriers:[10]

$$R_{14,23} = \frac{h}{e^2}\left(\frac{1}{T} - \frac{1}{b}\right). \tag{1}$$

In the absence of interedge channel scattering, the circulating and the transmitted edge channels remain decoupled and because each of the selectively populated edge channels is transmitted with a probability 1 for integer filling factors (complete adiabatic transport), the transmission $T = g$. In this case, the arrangement of two barriers behaves like a single barrier. In the opposite limit, where there is strong scattering (equilibrated transport) the system is characterized by a series resistance across both barriers and the total resistance is doubled. Both limiting cases have been demonstrated experimentally,[7] and consequently, the resistance plateau value $R_{14,23}$ is a sensitive measure of the interedge channel scattering rate.

The measurements are carried out in a ^{3}He-^{4}He-dilution refrigerator at $T = 100$ mK. The four-point resistances were obtained by standard lock-in techniques using low current levels (10 nA) and frequencies (< 15 Hz). Two devices with spacing $d = 20$ and 200 μm between the gates are compared in Fig. 1(b). Both samples are fabricated from the same heterojunction. In Fig. 1(b) data are shown for fixed $b = 2$ whereas g is varied. For the case of $d = 20$ μm we observe nearly complete adiabatic transport; the resistance plateau around $g = 1$ is close to $h/2e^2$. This means that the net current flow within the edge channel picture is highly spin polarized. For $d = 200$ μm the plateau is shifted towards higher resistance indicating that spin-flip interedge channel scattering processes have taken place.

A simple analysis, following a model of MacDonald,[11] allows us to extract the equilibration length directly from the measured resistance plateau values. In the analysis we trace the possible paths of an electron which has been transmitted through the left gate. For the sake of simplicity we discuss the analysis for $b = 2$ and $g = 1$.

At point A an edge channel electron enters the region between the gates via the outer channel. The probability of finding such an electron, carrying a net current, in the outer-edge channel at A is therefore 1. On the way from $A(x = 0)$ to $B(x = d)$ a fraction of these electrons is transferred into the inner-edge channel by interedge channel scattering. Assuming a constant number of interedge channel scattering events per unit length ($l_{\rm eq}^{-1}$), the probability of finding an electron in the outer-edge channel (between A and B) can be obtained by solving the system of rate equations:

$$P(x) = \frac{1}{2} + \frac{1}{2}\exp\left(-\frac{2x}{l_{\rm eq}}\right). \tag{2}$$

Here $l_{\rm eq}$, the equilibration length, corresponds to the distance an electron travels between two interedge channel scattering events.

At B only those electrons which are still in the outer-edge channel are transmitted and we define the transmission probability on the first pass as $T_1 = P(x = d) \equiv P$. The scattered ones $(1 - P)$ move from B to C without further scattering (only one edge channel present). From C to D some of the electrons are scattered into the outer-edge channel and these constitute the fraction of electrons which are transmitted at D and therefore reflected by the whole double-barrier structure. The fraction of electrons which remain in the inner channel have a chance of being transmitted at B on their second attempt if they are scattered on their way from A to B into the outer-edge channel. The probability of an electron being transmitted on the second pass is therefore $T_2 = P(1 - P)(1 - P)$, and the probability for an electron being transmitted on the ith attempt is $T_i = P^{2i-3}(1 - P)^2$.

The total probability for an electron being transmitted across both barriers is $T = \sum_{i=1}^{\infty} T_i$ and summing up all the T_i gives $T = 2P/(1 + P)$.[12] We therefore obtain for the resistance,

$$R_{14,23} = \frac{h}{e^2}\left[1 + \exp\left(-\frac{2d}{l_{\rm eq}}\right)\right]^{-1}. \tag{3}$$

Using Eq. (3) the equilibration lengths can be directly deduced from the measured resistance plateau values. For the gate finger spacing $d = 20$ μm we find $l_{\rm eq} = 440$ μm whereas for $d = 200$ μm, $l_{\rm eq} = 940$ μm. Within the experimental error (accuracy of the measured resistance 3%), however, both equilibration lengths are the same, because for $d = 20$ μm the measured resistance is close to the complete adiabatic value $h/(2e^2)$ and hence a small uncertainty causes a drastic change in $l_{\rm eq}$ [$l_{\rm eq} = \infty$ for $R_{14,23} = h/(2e^2)$]. We have obtained equilibration lengths well above 100 μm for a variety of samples. Using a device geometry similar to the one used in Ref. 6 and analyzing the anomalous Hall voltage[13] along the paths of Ref. 14 gives similar results.

The inset of Fig. 1(b) shows the temperature dependence of the inverse equilibration length which is proportional to the spin-flip scattering rate. At temperatures below 250 mK, we observe a saturating scattering rate; at higher temperatures the inverse equilibration length increases significantly.

As pointed out above, the situation $b = 2$ and $g = 1$ is a very special one. The electrons in the two available edge channels have opposite spin orientation. The levels from which they are formed are separated energetically by the spin gap $g\mu_B B \approx 90$ μeV at $B = 3.9$ T. μ_B is Bohr's magneton and g is the nonenhanced g factor of ≈ 0.4, which has been experimentally determined for similar heterostructures.[15,16] For a Landau gap of 3.2 meV a spatial separation of 780 Å between two neighboring (spin-degenerate) edge channels has been reported.[17] This value has been obtained from an analysis of the current dependence[18] of the resistance plateaus for $b = 4$ and $g = 2$ carried out on a similar sample. Assuming the same linear edge electric field[17] $F_{\rm edge} = \hbar\omega_c/(e\,780\text{ Å})$ for a smooth confining potential allows a rough estimate of 20 Å for the spatial separation δy between the two spin-split edge channels. A large overlap between the spin-up and spin-down edge channels is therefore expected since the spatial extent of the wave function is of the order of the magnetic length $l_c = \sqrt{\hbar/(eB)}$ [130 Å at $B = 3.9$ T, Fig. 1(b)] thus making adiabatic transport more unlikely.

Three physical processes are thought to be responsible for spin-flip scattering: the magnetic impurity scattering, the spin-orbit interaction, and the hyperfine interaction. Since the presence of such magnetic impurities is unlikely in our state-of-the-art MBE material, we rule out magnetic impurity scattering. For the case of the hyperfine interaction the electronic spin flip is accompanied by a nuclear spin flip and therefore the total spin is conserved. The lack of inversion symmetry both of the heterojunction interface (normal to the plane of the 2DEG) and of the crystal zinc-blende structure of GaAs itself[19,20] gives rise to a net nonvanishing electric field. This electric field causes a finite magnetic field in the moving frame of reference of an edge channel electron. For this mechanism of spin-orbit coupling the spin flip of an electron is accompanied by a change in the electronic momentum due to scattering. In the following we assume that spin-orbit interaction is more relevant for spin relaxation than hyperfine interaction.[21]

We estimate equilibration lengths assuming that spin-orbit interaction is the dominant source of spin-flip scattering. The underlying detailed theory will be published elsewhere.[22] The starting point is the Hamiltonian $\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_1$ where

$$\hat{\mathcal{H}}_0 = \frac{p^2}{2m} + V(y). \tag{4}$$

$\hat{\mathcal{H}}_0$ describes the motion of the 2DEG electron in a confining potential in the y direction and in the presence of a perpendicular magnetic field B with $\mathbf{p} = -i\hbar\nabla - e\mathbf{A}$ where $\mathbf{A}$ is the vector potential. The perturbative part is given by[20,23]

$$\hat{\mathcal{H}}_1 = \alpha\mathbf{n}(\mathbf{p}\times\boldsymbol{\sigma}) + \beta\boldsymbol{\sigma\kappa} + \tfrac{1}{2}g\mu_B B\sigma_z. \tag{5}$$

The first term describes the spin-orbit coupling due to the electric field at the heterojunction interface. The coupling constant α depends on this electric field, $\boldsymbol{\sigma}$ are the Pauli matrices, and $\mathbf{n}$ is the normal to the plane of the 2DEG, pointing in the z direction. The second term arises from the lack of inversion symmetry in the elementary cell of the bulk GaAs crystal with the coupling constant β; $\boldsymbol{\kappa}$ depends on the crystallographic orientation. The third term describes the well-known Zeeman energy. For an orientation of the sample edges along [001] $\mathbf{n}(\mathbf{p}\times\boldsymbol{\sigma}) = (p_x\sigma_y - p_y\sigma_x)$ and $\boldsymbol{\kappa} = (-p_x, p_y, 0)$. The parameter β can be expressed in terms of $\langle p_z^2\rangle$, the effective mass m, and the GaAs band gap E_g: $\beta = 0.035\langle p_z^2\rangle/m^{3/2}(2E_g)^{1/2} = 2.6\times10^3$ m/s.[23–25] Using the spin-degenerate eigenstates $\exp(ikx)\varphi_k(y)$ of $\hat{\mathcal{H}}_0$ to treat $\hat{\mathcal{H}}_1$ perturbatively, we find the energy splitting at any value of k to be

$$2\Delta = \sqrt{(g\mu_B B)^2 + (2mv_x)^2(\alpha^2+\beta^2)}, \tag{6}$$

with v_x the group velocity of an edge channel electron. Since α is smaller than β (Ref. 26) we neglect α for the numerical calculation of the internal magnetic field $2mv_x\sqrt{\alpha^2+\beta^2}/(g\mu_B)$ arising from the spin-orbit interaction. For $v_x = F_{\text{edge}}/B = 1\times10^4$ m/s with $B = 3.9$ T we obtain a value of about 0.9 T for this internal magnetic field. This internal field, lying in the plane of the 2DEG, gives rise to a local g-factor enhancement at the edge. If α is neglected all the results are valid also for a [011] orientation of the edges.

The presence of spin-orbit terms in the Hamiltonian allows transitions between spin-split edge channels in the presence of any scattering mechanism which changes the wave vector along the boundary. Elastic transitions between the edge channels not only change $k = -y/l_c^2$ (with the center coordinate y) but also the group velocity. In the Born approximation, we obtain for the equilibration length, with i indicating the initial and f the final state,

$$l_{\text{eq}} = l_0/|\chi_+^\dagger(k_i)\chi_-(k_f)|^2. \tag{7}$$

Hence the equilibration length is inversely proportional to the spinor overlap and the parameter l_0 corresponds to the scattering length of the spin-free problem. Assuming $\delta v_x = |v_{x,i} - v_{x,f}| \ll v_x$ the spinor product can be written as

$$\chi_+^\dagger(k_i)\chi_-(k_f) = \frac{\delta v_x}{v_x}[g\mu_B Bmv_x\sqrt{\alpha^2+\beta^2}/(4\Delta^2)]. \tag{8}$$

Calculating l_0 for long-range scattering according to Ref. 27 results in $l_0 = 4$ Å. The large value of the equilibration length observed must therefore be due to a small spinor overlap in Eq. (7). To estimate δv_x we take a parabolic confinement potential with a confinement frequency of $\Omega_0 = 7.8\times10^{11}$ s^{-1} which directly follows from our estimated edge channel group velocity. Using $\delta v_x/v_x = l_c^2 m\Omega_0^2\delta y/\hbar v_x$ we obtain an equilibration length of $l_{\text{eq}} = 160$ μm. This is of the order of the equilibration length observed experimentally.

Our theoretical approach is based on a single-particle picture. The long equilibration length is a consequence of the mechanism of spin-orbit scattering itself and also of a local g-factor enhancement at the edge of the sample. In the bulk of a 2DEG it is known that electron-electron interaction leads to a drastic enhancement of the g factor[28] if the Fermi energy is located between spin-split levels because then the electron system is spin polarized. At the edge of the sample there is always a spin polarization due to the presence of the edge channels and it is still an open question whether such a mechanism causes an enhancement of the local g factor and hence an increase of the spatial separation of spin-polarized edge channels. Such a local enhancement at the edge and the accompanied reduction of the wave-function overlap could reduce the spin-flip scattering rate. Our experiments do not indicate whether such a mechanism plays a role; we want to point out, however, that the long equilibration lengths observed experimentally are consistent with a picture solely involving spin-flip scattering via spin-orbit interaction, not taking into account electron-electron interaction.

In summary we have determined equilibration lengths for scattering between two spin-split edge channels where the only possible interedge channel scattering mechanism is spin-flip scattering. Lengths up to ~ 1 mm are found by using a basic model for the determination of the equilibration length from the measured resistance plateau value. A theory considering spin-flip interedge channel scattering via spin-orbit interaction is presented. The model leads to a local g-factor enhancement at the edge and is appropriate to explain the experimentally observed equilibration length.

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*Permanent address: Institute of Microelectronics, Academy of Science of the USSR, 142 432 Chernogolovka, Moscow District, U.S.S.R.

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