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Risk of Bankruptcy and the Modigliani-Miller theorem in a general equilibrium model of socially responsible investing☆

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ABSTRACT

We build on the Arrow–Debreu general equilibrium model by Arnold (2023) which features SRI in the form of portfolio preferences. Using assumptions from the classical finance literature, that model is modified such that it allows for bankruptcies. Firm capital inputs are shown to be entirely independent of debt levels, which constitutes the standard corporate finance Modigliani-Miller theorem. As long as debt alterings create no new spanning opportunities and do not destroy old ones, this result extends to a general equilibrium version, implying constant consumption of all individuals. If financial markets are not complete, higher debt may create new spanning opportunities and thus enhance welfare. This result is particularly relevant if individuals constrain the set of firms the assets of which they are willing to hold due to SRI.

1. Introduction

The question whether non-financial investment criteria influence economic activity, be it via individual rates of return or through different firm behavior, has been widely discussed in the recent literature on Socially Responsible Investing (SRI). Despite some tendency towards showing reduced returns of – primarily – green, i.e., ecologically friendly investment opportunities (see the literature review of Cheong and Choi (2020)), there does not seem to be a clear consensus yet.

From a theoretical viewpoint, social responsibility criteria may lead to better payoffs for non-responsible individuals due to costly innovation by responsible firms (Oehmke & Opp, 2021). However, a varied investment attitude may also turn out neutral for the economy because that innovation would have to be carried out by firms that responsible individuals shirk (Gollier & Pouget, 2022). Lastly, the ground-breaking work of Heinkel et al. (2001) shows that a critical mass of responsible investors can shift the market in favor of more responsibility.

Empirically, the picture is much the same. One can find various studies documenting negative abnormal returns of responsible assets (see Bolton and Kacperczyk (2021) and Flammer (2021)), equality of returns (Andersson et al., 2016; Hamilton et al., 1993) or outperformance by responsible assets (Edmans, 2011; Kempf & Osthoff, 2007).

In light of the above disagreements among different researchers, we go back to a very basic yet rich modeling approach, namely general equilibrium. The model is due to Arnold (2023). It is a generalization of the general equilibrium model with different states of nature in the second of two time periods, originally due to Arrow (1964) and Debreu (1959), augmented to include SRI. There, SRI

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turns out neutral if four conditions hold. Firstly, asset tastes and consumption have to enter utility separately. Secondly, there has to be at least one neutral (laxly speaking, an "irresponsible") individual, drawing no utility from the sheer act of holding any asset whatsoever. While technically necessary, this assumption is not particularly strong. Rather, the contrariant case of requiring every individual within a potentially large society to employ some kind of pro-sociality would yield a much stronger condition. Thirdly, responsible individuals must show satiation in SRI, which prevents them from financing infinite long positions of favorable firms' assets by shorting unfavorable ones. Lastly, each individual must experience a very strict version of financial market completeness (*Spanning With Assets with No Social returns*, in short *SWANS*). This is given as long as the individual can allocate consumption to any specific state of nature without using either favorable or unfavorable assets, that is, using only assets she is *indifferent towards*.

In Arnold (2023), budget constraints are built based on the assumption of no risk of bankruptcy. This is modeled by assuming that each firm offering an exogenous supply of bonds b_{jl} makes a sufficient nominal profit $p_{ls}y_{jls}$ to cover all resulting obligations in any possible state of nature $s \in S$, i.e., $p_{ls}y_{jls} \geq b_{jl}, s = 1, \dots, S$.[1] We drop this assumption and show that an adequate redefinition of budget constraints and firm values leaves the subsequent analysis unaffected.

It is apparent that, in the real world, over-indebtedness and bankruptcy are an issue. The only way of fully and inevitably circumventing it would be to oblige firms to finance all of their capital needs through equity, i.e., stocks. Hellwig (1976, p. 1) already noted this in an early version of his ground-breaking work on limited liability: "If the debt-equity ratio is large enough, there will always be a positive probability of bankruptcy". With the number of people (directly) holding stocks at all still below 20% even in industrialized countries,[2] it becomes apparent that, given households shift wealth intertemporally, they must do so via debt contracts (which, in turn, are ultimately issued by some firm). So excluding the possibility of bankruptcy is simply impossible to reconcile with allowing all individuals to transfer wealth over time.

To arrive at validity of the model even with risk of bankruptcy, two (reasonable) assumptions need to be additionally employed: limited liability of shareholders as well as no transaction costs in consequence of a firm going bankrupt. Both are common in related work, comprising Stiglitz's (1969) and those following it. These assumptions are focal if one wants to arrive at the result of Modigliani and Miller (1958) that the capital structure of a firm is irrelevant for its value (cf. Rubinstein (2003), p. 11).

Previous (already very early) work on this subject has shown that the above conclusion can be reached via two main strategies: using a mean–variance-framework or assuming a full set of *Arrow–Debreu securities* (ADS, cf. Stiglitz (1969), pp. 788–792). We proceed to show that any version of financial market completeness will do, i.e., the ability to synthesize portfolios that transfer wealth into any state of nature directly is sufficient. This formulation is somewhat more general as it allows the way financial market completeness is achieved to change when comparing different economies. An ADSs can be formed by building a certain sub-portfolio of assets. The composition of these sub-portfolios may be different across two economies.

Completeness of financial markets itself is crucial in order to guarantee a Pareto-efficient allocation of resources. In the presence of SRI, however, individuals may artificially constrain themselves in the degree of market completeness they experience. The most prevalent instance of this state of affairs is *negative screening*, which is currently the third-most popular strategy for sustainable investments with a volume of 3840 billion USD for the USA, Canada, Australia and Japan taken together (cf. Global Sustainable Investment Alliance (2022), p. 13). The term refers to a refusal to buy stocks or bonds of certain firms considered irresponsible by the individual. If this kind of behavior prevents her from holding an asset that is vital to financial market completeness (i.e., if *SWANS* is violated), the latter is no longer given for her. But by over-indebting itself, a different firm she is willing to hold assets of may fundamentally change its payoff structure and thereby re-establish a fully spanned state-space.

The theorems in this paper always compare two different versions of an economy with each other that differ only in terms of individual preferences or corporate financial capital structures. While a typical strategy to show equivalence between two economies in terms of endogenous real variables is to assume a complete financial market, a slight generalization can be made. Namely, one can instead assume that all market imperfections are the same for both economies. Naturally, this includes zero imperfections as a special case.

The rest of the paper is organized as follows. Section 2 traces the model of Arnold (2023) with exogenous bond supply and over-indebtedness. Section 3 introduces the firm's decision process on capital formation and bond emissions, ultimately reaching the Modigliani-Miller theorem of corporate finance. In Section 4, we generalize this result to irrelevance of financial structures for economic activity of the entire economy in the absence of SRI. Section 5 re-introduces SRI into this discussion. In Section 6, we discuss conditions under which excessive debt of some firms may increase welfare. Finally, Section 7 offers a discussion of the results.

2. Exogenous bond supply

The model is built around the portfolio decisions of $i = 1, \dots, I$ individuals. There is a single good in $t = 0$ with a price of p_0. Uncertainty is depicted as in $t = 1$, one out of S states of nature materializes. There are $l = 1, \dots, L$ goods priced at p_{ls}, respectively, in all states $s = 1, \dots, S$ in $t = 1$. After $t = 1$, the model ends. Each individual is endowed with y_{i0} units of the $t = 0$-consumption good. $t = 1$-goods are produced by J_l firms each. For their production, firms invest into physical capital k_{jl}, which can be obtained one-to-one from the $t = 0$-consumption good. The firm issues bonds b_{jl} priced at R_{jl}. Each individual i has an initial share $\bar{\theta}_{ijl}$ in firm (j, l)

[1] The indices l and j refer to goods and the firms producing them, respectively. Notation will be properly introduced in Section 2 and may be taken from the notation table.

[2] Current numbers encompass 17.6% in Germany (cf. Fey (2024)) and 15.2% in the USA (cf. Maranjian (2023)).

that determines how much of their bond emission profits she is entitled to and how much of its capital she must finance. Naturally, each share must belong to someone, so $\sum_{i=1}^{I} \bar{\theta}_{ijl} = 1, j = 1, \dots, J_l, l = 1, \dots, L$. Capital translates into output in $t = 1$ according to state-dependent production functions $y_{jls} = f_{jls}(k_{jl})$. Furthermore, the consumer can buy bonds, where individual holdings are called a_{ijl}, change her shareholdings to θ_{ijl} at a price determined by firm values v_{jl}, and trade assets in zero net supply (securities) z_{im} with the other individuals at price q_m. Taking all of this into consideration, i seeks to maximize her overall utility $U_i(u_i(\boldsymbol{c_i}, \boldsymbol{k}), \boldsymbol{\theta_i}, \boldsymbol{a_i}, \boldsymbol{k}, \boldsymbol{v})$, which depends on consumption utility $u_i(\boldsymbol{c_i}, \boldsymbol{k})$ and portfolio choice of stocks $\boldsymbol{\theta_i}$ and bonds $\boldsymbol{a_i}$ in a separable way.[3] Dependence of u_i on physical capital formation $\boldsymbol{k}$ depicts production externalities. u_i has the usual property of positive, decreasing marginal utility of consumption in every good. In applications where SRI does not play a role, U_i and u_i coincide. As the two rationales for suffering from externalities and having (dis-)tastes for assets are allowed to coexist, one can speak of a consequentialist and a non-consequentialist utility component. The former means that individuals suffer from the externality while the latter implies "warm glow" and "cold prickle" from certain asset holdings of favorable and unfavorable firms, respectively (see Dangl et al. (2024)).

The assumption of separability of overall utility in consumption on the one hand and asset holdings on the other is common but strong. It is not just employed by Arnold (2023) in his proof of irrelevance of SRI, but also, e.g., by Dufwenberg et al. (2011) in their proof of irrelevance of other-regarding preferences.[4] Consumption and portfolio formation only interact insofar as the latter finances and determines the former. It seems intuitive that the well-being derived from market goods' consumption is not directly influened by an individual's assets. However, the range of admissible utility functions is thus gravely limited.

Let the number of corporate bonds $b_{jl} \geq 0$ issued by firm (j, l) be given exogenously, where no restriction on the range of values it may take on is imposed. Each of these bonds promises, as in Arnold (2023), to pay one unit of income in the subsequent period. However, this promise may turn out to be hollow: Given that in some state(s) of nature the corporation that issues this debt goes bankrupt, the payoff of one such bond will turn out strictly less than one. If this is the case, the price of such a bond from a firm with positive risk of bankruptcy obviously is below that of a risk-free counterpart. Taking the resulting fair pricing of firms' bonds into account, budget constraints from Arnold (2023) can simply be rewritten. Proceeding in this fashion, the period $t = 0$-budget constraint reads

$$p_0(c_{i0} - y_{i0}) + \sum_{l=1}^{L} \sum_{j=1}^{J_l} [\bar{\theta}_{ijl}(p_0 k_{jl} - R_{jl} b_{jl}) + (\theta_{ijl} - \bar{\theta}_{ijl}) v_{jl} + R_{jl} a_{ijl}] + \sum_{m=1}^{M} q_m z_{im} \leq 0, \tag{1}$$

where the reduced value of some (or, potentially, all) firm-issued assets is taken into account by bond prices $R_{jl} \leq R$. We can be assured that those bond prices will coincide with the price of a safe asset R if and only if firm (j, l) does not go bankrupt in any state of nature s. Bond prices obey

$$R_{jl} = \begin{cases} = R & \text{if } p_{ls} y_{jls} \geq b_{jl},\ s \in S \\ < R & \text{if } \exists s \in S : p_{ls} y_{jls} < b_{jl} \end{cases},$$

where we slightly abuse notation in defining the set $S = \{1, \dots, S\}$. From this we can infer that the introduction of bankruptcy risk will, ceteris paribus, relax the first $(t = 0)$ budget constraint somewhat for investors of a firm while it tightens it for initial shareholders. Turning to the $t = 1$-budget constraint, one now has to consider additional nonlinearity. Namely, for each firm (j, l) that i invests into, the possibility that her shares of that firm become worthless needs to be taken into account. To do so, we introduce the concept of limited liability of shareholders into the model: If $p_{ls} y_{jls} < b_{jl}$ holds for firm (j, l) of which i holds positive shares $\theta_{ijl} > 0$, she is not obliged to make up for this firm's over-indebtedness with her private wealth. Such assumptions are quite common in financial economics (see, for example, Magill and Quinzii (1996), pp. 401–407). Rather, shares in this corporate become worthless while creditors obtain a fraction of its bankruptcy estate that is proportional to the number of bonds they bought from this very firm relative to its total issuance. Considering the above, the budget constraints for date $t = 1$ can be written as

$$\sum_{l=1}^{L} p_{ls} c_{ils} - \sum_{l=1}^{L} \sum_{j=1}^{J_l} [\theta_{ijl} \max(p_{ls} y_{jls} - b_{jl}, 0) + \alpha_{ijls}] - \sum_{m=1}^{M} x_{ms} z_{im} \leq 0, \ \ s = 1, \dots, S, \tag{2}$$

where the bond-payoff

$$\alpha_{ijls} = \min\left(a_{ijl}, \frac{a_{ijl}}{b_{jl}} p_{ls} y_{jls}\right) = \begin{cases} a_{ijl} & \text{if } p_{ls} y_{jls} \geq b_{jl} \\ \frac{a_{ijl}}{b_{jl}} p_{ls} y_{jls} & \text{if } p_{ls} y_{jls} < b_{jl} \end{cases}, \ \ s = 1, \dots, S \tag{3}$$

expresses the seminal assumption described above that losses in bond value are borne proportionately by all bondholders (see also Sabarwal (2000)). In order to be able to apply the analysis in Arnold (2023), state prices r_s need to be determined. The rationale for this is that, with risk of bankruptcy, we can no longer define firm values as "state price-weighted profits minus debt" because this difference may become negative while actually no firm can be anything worse than worthless. To correct this, one can either employ the maximum operator once again or (as done here) define a separate set of states without bankruptcy per firm.[5] We do so by specifying the sets S'_{jl} as the collection of those states s in which $p_{ls} y_{jls} \geq b_{jl}$ is satisfied for the specific firm (j, l) under

[3] Bold symbols refer to vectors of variables where entries vary by the omitted index, so $\boldsymbol{\theta}_i$, e.g., stacks θ_{ijl} over $j = 1, \dots, J_l$, $l = 1, \dots, L$.

[4] The separability notion of Dufwenberg et al. (2011) concerns own and other individuals' consumption bundles rather than own consumption and portfolios.

[5] What makes this approach more attractive is the flexibility and re-usability of the so-defined set which will be heavily made use of later.

consideration.[6] One may thus refer to this set as a given firm's set of solvency states. Using this definition, we obtain firm values obeying

$$v_{jl} = \sum_{s \in S'_{jl}} r_s (p_{ls} y_{jls} - b_{jl}), \; j = 1, \ldots, J_l, \; l = 1, \ldots, L. \tag{4}$$

The next equation necessary for pinning down state prices gives the cost of purchasing a safe asset. Naturally, it is just the sum of all ADSs' prices (see also Arnold (2023), p. 72):

$$R = \sum_{s=1}^{S} r_s, \tag{5}$$

but we must note that such an asset may not exist: If every firm goes bankrupt in some state, neither of them issues a safe bond. A sufficient condition to ensure existence of such an asset would be to assume at least one firm (j,l) that satisfies $p_{ls} y_{jls} \geq b_{jl}$ in every state $s = 1, \ldots, S$, or put differently: a firm with enough profits to service all of its debt in each possible state of nature, $\exists (j,l) : S'_{jl} = S$. This is, however, not a necessary condition, as the nonexistence of a safe corporate bond need not impinge on market completeness.[7] Payoff profiles may still be synthesized by means of security trade or purchase of stocks. A suitable combination of bonds from various firms is also possible. That is, while we may not obtain a single safe asset, the construction of a safe portfolio can still be unhindered. Ultimately, there is nothing that prevents individuals from creating a safe asset themselves, i.e., one of the M zero net-supply assets in $\boldsymbol{z}$ could be safe as well.

In fact, bankruptcy here may serve as a form of financial engineering. More precisely, a "securities innovation" can happen. The term is borrowed from Finnerty (1988), who describes it as the creation of new financial instruments. In our context, this means that new payoff profiles can be synthesized.[8] To see how a structure of payoffs unavailable before can be generated, imagine two firms, (j,l) and (j',l') with the same revenues $p_{ls} y_{jls} = p_{l's} y_{j'l's}, s = 1, \ldots, S$, perhaps because they produce the same good $l = l'$ with identical production structure $f_{jls}(k_{jl}) = f_{j'l's}(k_{j'l'}), s = 1, \ldots, S$, and will thus endogenously choose the same capital input $k_{jl} = k_{j'l'}$. Obviously, both stocks and bonds issued by one of these firms are perfect substitutes for financial products of the other if bankruptcy cannot happen. If, however, one of them instead raises so much outside capital that it cannot fully service its external debt in some states s, payoffs from both its stocks and its bonds will change, demarcating it somewhat from the more conservatively financed corporation. So the subset of the state-space spanned by the two firms under consideration alone has increased by over-indebting one of them, meaning that the risk of bankruptcy can indeed (but need not necessarily) help in achieving completeness of financial markets. In Sabarwal's (2000, cf. p. 2) words, the equilibrium allocation of an economy with bankruptcy may exert Pareto-dominance over an analogous one where bankruptcy is ruled out from the beginning. Note, however, that this may also affect state prices r_s, so the described positive effect could be mitigated or even reversed for some individuals.

We can compute the prices of each single firm's corporate bond as

$$R_{jl} = \sum_{s \in S'_{jl}} r_s + \sum_{s \notin S'_{jl}} \frac{p_{ls} y_{jls}}{b_{jl}} r_s, \; j = 1, \ldots, J_l, \; l = 1, \ldots, L. \tag{6}$$

This is the present value of all factually expected payoffs from that bond, which are below unity for states where the firm under consideration is insolvent. The price of a safe asset as taken from (5) is obtained as a special case for all firms that never go bankrupt.[9] Therefore, the possibility of bankruptcy in some (that is, a fixed subset of) states can serve as a means to make a single corporation's bonds unique as already seen in the example above. In exchange for this, however, the property of all bonds being perfect substitutes is lost. In a similar vein as in Stiglitz's work on the Modigliani-Miller theorem, this implies higher interest rates being paid (via their bonds) by firms with higher risk of bankruptcy (cf. Stiglitz (1969), p. 788). Finally, unchanged compared to the original setup by Arnold (2023), security prices are still obtained by weighting payoffs with according state prices, i.e.,

$$q_m = \sum_{s=1}^{S} r_s x_{ms}, \; m = 1, \ldots, M. \tag{7}$$

Upholding the same assumptions, each of the subsequent analyses conducted by Arnold (2023) remains valid. To illustrate this point, we relate equilibria with different debt levels in Section 4. Before doing so, it seems adequate to discuss an endogenous optimum choice of leverage.

[6] Naturally, there is one such set for each firm (j,l), i.e., $\sum_{l=1}^{L} J_l$ sets of this kind in total. Some of these may constitute the empty set or be identical to each other. All of them are subsets of S.

[7] Note further that market completeness need not be given in order for state prices to exist.

[8] Finnerty (1988) distinguishes this and two further kinds of financial engineering. The other two encompass reduced transaction costs as well as more creative management strategies for debt or cash. Both are incompatible with our modeling framework where transaction costs are absent and any way of financing is available to everyone.

[9] An alternative way of writing this is $R_{jl} = \sum_{s=1}^{S} r_s \frac{\alpha_{ijls}}{\alpha_{ijl}}, \; j = 1, \ldots, J_l, \; l = 1, \ldots, L$, where dependence on i cancels out.

3. Endogenous bond supply

Within the setting of a general equilibrium model, it may seem unreasonable to assume states with bankruptcy. All risks can be perfectly calculated as the number of states as well as the nominal revenues conditional on these states are known. So nothing prevents firms from planning farsightedly enough to avoid bankruptcies given the states $s = 1, \ldots, S$. Hence, some motivation for allowing bankruptcy to happen is in order. Simply obliging firms to raise outside capital to an extent which leads them to be over-indebted in some of the possible states of nature is somewhat artificial.[10] A good intuition as to why financial distress is usually avoided is given by Gilson (1989). He argues that managers of an insolvent firm bear large personal costs such as a loss in reputation and more difficult odds in the labor market, possibly mitigated somewhat by a fondness for leisure. Clearly, such long-term effects cannot be incorporated properly into the two-period framework considered here. We could merely ascribe a disutility of bankruptcy to the largest stockholder of each (j,l). If i has the absolute majority in that firm ($\theta_{ijl} > 0.5$) and if this is then translated into equivalent voting power, bankruptcy would be unconditionally avoided. If shares are spread out too far ($\theta_{ijl} \leq 0.5$), on the other hand, preventing bankruptcy is partly out of her control. If the latter happens, she ends up with a lower utility. Other than that, nothing changes.

We model an endogenous choice of debt b_{jl} by each firm (j,l) in order to analyze whether insolvency in some states may actually be the result of rationally optimal behavior. Our goal is to arrive at a version of the result due to Modigliani and Miller (1958) that a firm's capital structure is irrelevant for its value.[11] Note that, due to shareholders' obligations to partake in the costs of capital and their claim to the proceeds from bond emissions, what needs to be invariant is indeed not the firm value v_{jl} per se, but rather firm value less the aforementioned costs. This is precisely the shareholder value (SV):

$$SV_{jl} = v_{jl} - (p_0 k_{jl} - R_{jl} b_{jl}). \tag{8}$$

SV gives the true value firm (j,l) has for investor i considering all costs and benefits associated with owning one share of it.

3.1. Shareholder unanimity

Before moving on to the analysis of how SV is maximized, we must first ascertain whether SV maximization is indeed the goal of shareholders. Establishing that the budget sets of all initial shareholders in a firm (j,l), i.e., all i with $\bar{\theta}_{ijl} > 0$, expands as SV increases, is sufficient for the proof of shareholder unanimity – even in the presence of SRI – given by Arnold (2023, pp. 74–75) to go through without any modification.

In order to be able to comment on the budget set of i, we need to bring all she can afford into comparable terms. Therefore, we start with the BC of $t = 1$ which, for each state, is weighted using the adequate state price. Summing over all states after having applied this weighting delivers

$$\sum_{s=1}^{S} r_s \sum_{l=1}^{L} p_{ls} c_{ils} - \sum_{s=1}^{S} r_s \sum_{l=1}^{L} \sum_{j=1}^{J_l} [\theta_{ijl} \max(p_{ls} y_{jls} - b_{jl}, 0) + \alpha_{ijls}] - \sum_{s=1}^{S} r_s \sum_{m=1}^{M} x_{ms} z_{im} \leq 0.$$

Minor rearrangements yield

$$\sum_{l=1}^{L} \sum_{j=1}^{J_l} \left[\theta_{ijl} \sum_{s=1}^{S} r_s \max(p_{ls} y_{jls} - b_{jl}, 0) \right] \geq \sum_{s=1}^{S} r_s \sum_{l=1}^{L} p_{ls} c_{ils} - \sum_{l=1}^{L} \sum_{j=1}^{J_l} \sum_{s=1}^{S} r_s \alpha_{ijls} - \sum_{m=1}^{M} z_{im} \sum_{s=1}^{S} r_s x_{ms},$$

where the asset pricing Eqs. (4) through (7) are applicable. To see how, note that in the above equation we have

$$\sum_{s=1}^{S} r_s \max(p_{ls} y_{jls} - b_{jl}, 0) = \sum_{s \in S'_{jl}} r_s (p_{ls} y_{jls} - b_{jl}) = v_{jl}$$

according to (4),

$$\sum_{s=1}^{S} r_s \alpha_{ijls} = \sum_{s \in S'_{jl}} r_s a_{ijl} + \sum_{s \notin S'_{jl}} r_s \frac{a_{ijl}}{b_{jl}} p_{ls} y_{jls} = a_{ijl} R_{jl}$$

due to (6) and

$$\sum_{s=1}^{S} r_s x_{ms} = q_m$$

immediately by (7). Thus, we know that

$$\sum_{l=1}^{L} \sum_{j=1}^{J_l} \theta_{ijl} v_{jl} \geq \sum_{s=1}^{S} r_s \sum_{l=1}^{L} p_{ls} c_{ils} - \sum_{l=1}^{L} \sum_{j=1}^{J_l} a_{ijl} R_{jl} - \sum_{m=1}^{M} q_m z_{im} \tag{9}$$

[10] The counterargument that the firm generates a free revenue in $t = 0$ in return has no bite: Note that they gain less income via bond emission at a price of R_{jl} instead of R whereas the repayment cost associated with it in all non-bankruptcy states remains the same.

[11] As Rubinstein (2003, p. 7) notes, this central result in financial market theory is actually due to John Burr Williams who, in his book *The Theory of Investment Value* (1938, p. 73), described the underlying state of affairs under the name of the "Law of the Conservation of Investment Value".

has to hold. To further work with the BC for $t = 0$, rewrite (1) as

$$p_0(c_{i0} - y_{i0}) + \sum_{l=1}^{L} \sum_{j=1}^{J_l} \left\{ -\bar{\theta}_{ijl} \left[v_{jl} - (p_0 k_{jl} - R_{jl} b_{jl}) \right] + R_{jl} a_{ijl} \right\} + \sum_{m=1}^{M} q_m z_{im} + \sum_{l=1}^{L} \sum_{j=1}^{J_l} \theta_{ijl} v_{jl} \leq 0, \tag{10}$$

where SV according to (8) is explicitly singled out and the left-hand value of (9) appears. As the latter is known to be no less than the RHS of (9), we can replace it with just that without violating the weak inequality in (10). After doing so, the inequality gravely simplifies to

$$p_0 c_{i0} + \sum_{s=1}^{S} r_s \sum_{l=1}^{L} p_{ls} c_{ils} \leq p_0 y_{i0} + \sum_{l=1}^{L} \sum_{j=1}^{J_l} \bar{\theta}_{ijl} SV_{jl}.$$

It thus becomes apparent that the upper limit of affordable "consumption value" increases with shareholder value SV_{jl} as long as $\bar{\theta}_{ijl} > 0$. Hence, in terms of consumption utility, each initial shareholder benefits from an increase in SV, which is in turn necessary to obtain shareholder unanimity with the goal of SV maximization. Without social responsibility concerns, it actually follows immediately. As can be seen in Arnold (2023, p. 74), it can also be guaranteed for utility functions displaying preferences that fit classification-based SRI while being independent of bond holdings. That is, we can assume for convenience that if firm (j,l) influences the utility of i, it does so via its market value v_{jl} scaled by her shareholdings θ_{ijl}. This implies a restriction towards non-consequentialist preferences. Alternatively, one can postulate that more than half of the shares of every firm are held by individuals that obtain no disutility from that k_{jl} and the resulting v_{jl} to ensure SV maximization. In the latter case, however, shareholder unanimity does not necessarily prevail.

The fact that investor preferences remain largely unspecified in the model as it stands provides further intuition for why, even without shareholder unanimity, the SV-maximizing k^*_{jl} can be expected to prevail. While all non-responsible shareholders of firm (j,l) agree on the optimum extent of its production, a SR one most likely will not agree with another. Depending on their preferences, their individually preferred capital input is some non-negative entry of the real line. Without an unambiguous solution concept, such as profit maximization, the chance that two SR investors agree on some k'_{jl}[12] is, thus, zero. With one-share-one-vote, the only possibility to obtain k'_{jl} is that one i that *cares about* the respective firm, i.e., gets utility or disutility from its assets, holds more shares alone than all individuals that are *indifferent towards* it taken together. In other words, k^*_{jl} is most likely the major minority found in the assembly.

To sum up, the channel of investor voice is effectively shut down in the model at hand. Broccardo et al. (2022), on the contrary, identify shareholder voting as an effective way to give SRI impact when social responsibility is widespread. While the absence of this channel may seem like a stark limitation of our model's validity, it serves to clearly disentangle that channel from another, which is passive investing. That is, this paper answers the question what portfolio shifts can achieve on their own.

Knowing that shareholders truly aim at maximizing SV is an important interim result. In what follows, this will serve as a cornerstone to analyze the effect of changing debt levels of the corporations both for themselves and the economy as a whole.

3.2. Optimizing shareholder value

Having shown that SV should indeed be maximized, the question arises of how exactly this is to be achieved. In other words, the vector of optimum capital inputs $\boldsymbol{k}^*$ needs to be determined. The SV of firm (j,l) in the no-bankruptcy-model is given by (cf. Arnold (2023), p. 73)

$$v_{jl} - (p_0 k_{jl} - R b_{jl}), \tag{11}$$

where

$$v_{jl} = \sum_{s=1}^{S} r_s (p_{ls} y_{jls} - b_{jl}), j = 1, \ldots, J_l, l = 1, \ldots, L.$$

In this benchmark-version of the model, bankruptcy is not an issue. The SV-maximizing choice of k_{jl} obeys

$$\frac{d[v_{jl} - (p_0 k_{jl} - R b_{jl})]}{d k_{jl}} \stackrel{!}{=} 0.$$

As firms' outputs are determined by a (state-dependent) production function with capital as the only input, $y_{jls} = f_{jls}(k_{jl})$, we denote the derivative of output with respect to capital, i.e., the marginal product of capital, as $f'_{jls}(k_{jl})$. We immediately obtain the first-order condition (FOC) for SV-maximization as

$$\sum_{s=1}^{S} r_s p_{ls} f'_{jls}(k_{jl}) = p_0. \tag{12}$$

[12] Note that we do not superimpose $k'_{jl} < k^*_{jl}$. If, for example, firm (j,l) is an oil producer but best-in-class, some environmentally motivated investors may advocate its expansion in order to take market shares from more polluting competitors. Others may try to shrink the overall sector and thus vote for capital input reductions of that firm.

Moving on to the more general case, we cannot simply solve $\partial[v_{jl} - (p_0 k_{jl} - R_{jl} b_{jl})]/\partial k_{jl} \overset{!}{=} 0$ with v_{jl} given by (4) because the expression cannot be differentiated at those s where $b_{jl} = p_{ls} y_{jls}$. However, it is easy to show that (12) holds even in the presence of arbitrary risks of bankruptcy once we express SV as a function of economic fundamentals, that is, non-financial variables. In the general setup, SV is given by (8). Inserting the definitions of firm value v_{jl} from (4) and bond prices R_{jl} from (6), we get

$$SV_{jl} = \sum_{s \in S'_{jl}} r_s \left(p_{ls} y_{jls} - b_{jl}\right) - \left(p_0 k_{jl} - \sum_{s \in S'_{jl}} r_s b_{jl} + \sum_{s \notin S'_{jl}} r_s p_{ls} y_{jls} \right),$$

which boils down to shareholder value as a function of economic fundamentals rather than the capital structure:

$$SV_{jl} = \sum_{s=1}^{S} r_s p_{ls} y_{jls} - p_0 k_{jl}. \tag{13}$$

Differentiating (13) with respect to k_{jl} and equating this derivative to zero again delivers (12). The fact that we obtain identical FOCs with and without risk of bankruptcy has two important implications:

- From the fact that the FOC when bankruptcy is possible does not depend on the states in which it actually occurs we can infer that this condition indeed looks the same for any level of debt. Therefore, we need not concern ourselves with the more tedious analysis of discrete, that is, non-marginal differences followed in the general equilibrium Modiglinai-Miller (MM) theorem's proof in Section 4.2.
- If we encounter the same prices for goods and states as well as production technologies in two economies where bankruptcy happens in one and does not in the other, we can infer that those two economies will also have firms choose identical capital stocks $k^*_{jl}, j = 1, \dots, J_l, l = 1, \dots, L$.

These implications will serve as an important lemma in showing that allowing for bankruptcy does not matter at all for real firm decisions and individual consumption allocations.

Lemma 1. *If state prices $r_s, s = 1, \dots, S$, goods prices $p_0, p_{ls}, l = 1, \dots, L, s = 1, \dots, S$ and production technologies $f_{jls}(k_{jl})$ are the same in two economies with different levels of debt $b_{jl}, j = 1, \dots, J_l, l = 1, \dots, L$, the optimal choice of capital k^*_{jl} will be the same as well $\forall (j, l)$.*

3.3. Modigliani-Miller theorem at the corporate level

Before employing any specific value for b_{jl}, we need to determine the range of values it may take on. Given that bond supply is a choice variable for firms where these firms themselves are owned by shareholders, the one central condition that needs to be satisfied is still SV maximization, now also with respect to b_{jl} rather than k_{jl}. The simplest approach that can be taken is to express SV via (13) and derive zero influence of b_{jl}. This is essentially the MM theorem at the corporate level.

In order to obtain from (13) that

$$\frac{dSV_{jl}}{db_{jl}} = 0,$$

in other words, that MM at the corporate level always holds true, we have to show that $dk_{jl}/db_{jl} = 0$. This is done in Appendix A.1.

The significance of this result rests in the fact that zero influence of b_{jl} on SV_{jl} holds for changes in bond emissions of arbitrary (that is, even any non-marginal) magnitude. One can see this as neither the set of solvency states S'_{jl} nor the amount of bonds issued b_{jl} appear in (13) at all.

Lemma 2. *Shareholder value is fully determined by economic fundamentals. So the corporate finance version of MM holds for every single firm.*

4. General equilibrium Modigliani-Miller theorems

This section analyzes whether an economy-wide version of MM holds in the model at hand. For this to be the case, we need to obtain not only constant SVs, which were shown in Section 3.3, but also unaltered consumption levels c_{i0} at $t = 0$, and c_{ils} of all goods $l = 1, \dots, L$ in all states of nature $s = 1, \dots, S$ at $t = 1$, for all individuals $i = 1, \dots, I$. From Lemma 2, we know that firms' capital stock choice is not influenced by any alteration in leverage. Thus, we can rest assured that unchanged consumption levels are in fact sufficient to conserve consumption utility levels $u_i(\boldsymbol{c}^*_i, \boldsymbol{k}^*) \forall i$ even in the presence of externalities. Of course, this does not yet imply that overall utility $U_i(u_i(\boldsymbol{c}_i, \boldsymbol{k}), \boldsymbol{\theta}_i, \boldsymbol{a}_i, \boldsymbol{k}, \boldsymbol{v})$ is indeed unaffected. Thus, although the subsequent analysis posits as a proof of neutrality of debt levels in the no-SRI case, the picture might change if SRI is relevant. SRI is ignored for now and will be returned to in Section 5. We introduce changes in the bond supply of firms and proceed to check if they can be neutralized. Intuitively, the analysis is similar to Stiglitz's (1974) in his seminal proof of the theorem. In his words, we show that "individuals can exactly 'undo' any financial policy undertaken by the firm" (Stiglitz, 1974, p. 859).

Clearly, from the fact that SV does not change, there should not be any direct effects of changing bond supply b_{jl} in a world of complete markets. There, via their initial shareholdings $\bar{\theta}_{ijl}$ and firm values v_{jl} of the leveraging firm (j, l), what consumers experience is a shift of consumption levels between time and/or states. When confronted with such a shift, individuals can simply buy or sell other assets that restore the former order. Things become less clear-cut for the more general case where bankruptcy is allowed to happen. There, changes in b_{jl} may also influence bond prices R_{jl} as well as the return on bonds α_{ijls}. Furthermore, creating new states with bankruptcy or shifting the pending repayments in those cases might impinge upon financial market completeness. MM would then not hold due to the fact that "bankruptcy changes the opportunity set facing a given individual" (Stiglitz, 1974, p. 862).

In order to assess the effect of arbitrarily large changes in debt issuances, we analyze the consequences of non-infinitesimal changes in bond supply Δb_{jl}. We skip dealing with marginal changes and instead simply note that the neutrality of "small" changes follows as a corollary from the neutrality of "large" changes.

The proofs in this and the following section comment on financial market completeness. In particular, they rely on the subset of the state-space that is spanned by all assets to remain the same when moving from one economy to another. This set is known as the *marketed subspace* (see Magill and Quinzii (1996), p. 76). The fact that it has to remain constant posits the central condition for all of the results derived in this section.

If the state-space is not of its maximum dimension S, markets are said to be incomplete. Only if every single state is available for a targeted consumption reallocation are financial markets complete. This implies that complete markets before and after a debt surge are a special case of a constant marketed subspace. All remarks on unaltered spanning opportunities may be skipped by readers prepared to employ the additional assumption of the entire state-space being spanned by, e.g., the zero net-supply securities $\boldsymbol{z}$.

4.1. General equilibrium MM: No bankruptcy

Arguing in favor of the validity of an economic MM theorem in the baseline model is straightforward: Given that markets are complete, in a world with no bankruptcy, the forced shift via additional debt issuings of firm (j,l) can be neutralized most simply via investing all issue proceeds back in bonds: It suffices for i to change bond holdings a_{ijl} of precisely that firm which increases its issuance from b_{jl} to $b_{jl}+\Delta b_{jl}$ by an appropriate share $\bar{\theta}_{ijl}\Delta b_{jl}$. The payoff profile of those bonds remains unchanged even if more of them are given out, directly via the assumption of no bankruptcy. Given that several firms change their debt levels from one version of the economy to the other, additional freedom in the choice of which corporation's assets to buy arises for the individual. Since no new consumption possibilities can arise due to Lemma 1, the restored $\boldsymbol{c}_i^*$ is indeed optimal $\forall i$.

We can assess neutralizability of debt emissions in a similar manner as Stiglitz (1974). To do so, start with the following notion of an equilibrium:

Definition 1. $\left((\boldsymbol{c}_i^*,\boldsymbol{\theta}_i^*,\boldsymbol{a}_i^*,\boldsymbol{z}_i^*)_{i=1}^I,\boldsymbol{k}^*,\boldsymbol{p}^*,\boldsymbol{v}^*,\boldsymbol{R}^*,\boldsymbol{q}^*,\boldsymbol{b}\right)$ is an equilibrium of the stock-and-debt-economy (ESDE) if, given prices $\boldsymbol{p}^*,\boldsymbol{v}^*,\boldsymbol{R}^*,\boldsymbol{q}^*$ and bond supplies $\boldsymbol{b}$ such that $p_{ls}y_{jls}\geq b_{jl}, s=1,\dots,S, j=1,\dots,J_l, l=1,\dots,L$,

- Consumption $\boldsymbol{c}_i^*$ and portfolios $(\boldsymbol{\theta}_i^*,\boldsymbol{a}_i^*,\boldsymbol{z}_i^*)$ maximize consumption utility u_i subject to (1) and (2) for all $i=1,\dots,I$.
- Capital choices $\boldsymbol{k}^*$ maximize SV for all $j=1,\dots,J_l$, $l=1,\dots,L$.
- Markets clear.

Assume we are at an equilibrium $\left((\boldsymbol{c}_i^*,\boldsymbol{\theta}_i^*,\boldsymbol{a}_i^*,\boldsymbol{z}_i^*)_{i=1}^I,\boldsymbol{k}^*,\boldsymbol{p}^*,\boldsymbol{v}^*,\boldsymbol{R}^*,\boldsymbol{q}^*,\boldsymbol{b}\right)$. Now let all firms vary their bond issuances by some Δb_{jl}, potentially taking the value 0 for some of them. The following theorem states that buying new bonds for the price of the additionally accruing issue proceeds restores optimal consumption $\boldsymbol{c}_i^*\forall i$ and, thus, equilibrium.[13]

Theorem 1. *If* $\left((\boldsymbol{c}_i^*,\boldsymbol{\theta}_i^*,\boldsymbol{a}_i^*,\boldsymbol{z}_i^*)_{i=1}^I,\boldsymbol{k}^*,\boldsymbol{p}^*,\boldsymbol{v}^*,R^*,\boldsymbol{q}^*,\boldsymbol{b}\right)$ *is an ESDE, then* $\left((\boldsymbol{c}_i^*,\boldsymbol{\theta}_i^*,\boldsymbol{a}_i^*+\boldsymbol{\Delta a}_i^*,\boldsymbol{z}_i^*)_{i=1}^I,\boldsymbol{k}^*,\boldsymbol{p}^*,\boldsymbol{v}^{**},R^*,\boldsymbol{q}^*,\boldsymbol{b}+\boldsymbol{\Delta b}\right)$*, where* $\boldsymbol{\Delta a}_i^*=\theta_i^*\odot\boldsymbol{\Delta b}$*, is also an ESDE as long as* $\boldsymbol{b}$ *and* $\boldsymbol{b}+\boldsymbol{\Delta b}$ *imply the same marketed subspace.*

Proof. Let $\boldsymbol{p}^*$, $\boldsymbol{r}^*$ and $\boldsymbol{q}^*$ be the same for both versions of the stock-and-debt-economy and let firm values $\boldsymbol{v}^*$ and $\boldsymbol{v}^{**}$ imply the same SV. Optimum capital choices are $\boldsymbol{k}^*$ in both economies due to Lemma 1. Thus, given the spanned state-space is unaltered, state prices $\boldsymbol{r}^*$ are, in fact, also the same,[14] which, of course, also extends to their sum R^*. Firm values indeed change from $\boldsymbol{v}^*$ to $\boldsymbol{v}^{**}$ such that a constant SV is warranted.

From the fact that $\left((\boldsymbol{c}_i^*,\boldsymbol{\theta}_i^*,\boldsymbol{a}_i^*,\boldsymbol{z}_i^*)_{i=1}^I,\boldsymbol{k}^*,\boldsymbol{p}^*,\boldsymbol{v}^*,R^*,\boldsymbol{q}^*,\boldsymbol{b}\right)$ is an equilibrium, we know that (1) has to hold with equality due to strictly positive marginal utility of consumption. Note that, as there is never any bankruptcy, we can replace R_{jl} with R for now. It suffices to show that the induced reallocation resulting from $\boldsymbol{\Delta b}$ can be neutralized. If we let individuals change nothing but their bond holdings, then $\Delta c_{i0}=0$ necessitates

$$\sum_{l=1}^{L}\sum_{j=1}^{J_l}\theta_{ijl}^*R^*\Delta b_{jl}=\sum_{l=1}^{L}\sum_{j=1}^{J_l}R^*\Delta a_{ijl}.$$

[13] Below, $\odot$ denotes the Hadamard product. The Hadamard product is defined for two vectors of the same dimension. The nth element of this product is simply the product of the two multiplied vectors' nth elements.

[14] Constant production decisions imply that the scarcity of consumption in all states remains unaltered. Thus, it seems reasonable to assume that goods prices, instruments for reflection of this scarcity, must also be identical.

So we know that total new bond purchases of i must satisfy

$$\sum_{l=1}^{L}\sum_{j=1}^{J_l} \Delta a_{ijl} = \sum_{l=1}^{L}\sum_{j=1}^{J_l} \theta^*_{ijl}\Delta b_{jl} \tag{14}$$

We are further interested in the change occurring in (2). Unsurprisingly, it turns out to be state-independent and is given by

$$\sum_{l=1}^{L}\sum_{j=1}^{J_l}\left[\theta^*_{ijl}(-\Delta b_{jl}) + \Delta a_{ijl}\right].$$

It can be solved for 0 to obtain that $\Delta c_{ils} = 0$ is also possible as long as

$$\sum_{l=1}^{L}\sum_{j=1}^{J_l} \Delta a_{ijl} = \sum_{l=1}^{L}\sum_{j=1}^{J_l} \theta^*_{ijl}\Delta b_{jl},$$

which is exactly (14) again. Thus, changing bond purchases according to precisely that equation will restore the former equilibrium allocation.

Furthermore, as argued above, no additional consumption possibilities arise. Therefore, no desire to change consumption can have arisen as well.[15]

It remains to show that markets still clear. To keep things simple, consider first the special case where all individuals $i = 1,\dots,I$ choose $\Delta a_{ijl} = \theta^*_{ijl}\Delta b_{jl}, j = 1,\dots,J_l, l = 1,\dots,L$. Stacking over j and l delivers $\boldsymbol{\Delta a}^*_i = \boldsymbol{\theta}^*_i \odot \boldsymbol{\Delta b}$, which is exactly the formulation in the above theorem. With it, we can easily see that total demand for bonds of each firm (j,l) equals

$$\sum_{i=1}^{I}\left(a_{ijl} + \Delta a_{ijl}\right) = \sum_{i=1}^{I} a_{ijl} + \sum_{i=1}^{I}\Delta a_{ijl} = b_{jl} + \Delta b_{jl}\sum_{i=1}^{I}\theta^*_{ijl},$$

which, due to stock market clearing $\sum_{i=1}^{I}\theta^*_{ijl} = 1, j = 1,\dots,J_l, l = 1,\dots,L$ of our starting point equilibrium, is just total bond supply $b_{jl} + \Delta b_{jl}$ of this very firm. Thus, markets clear. Without the specification we could so far merely conclude that additional demand and additional supply satisfy

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J_l}\Delta a_{ijl} = \sum_{l=1}^{L}\sum_{j=1}^{J_l}\Delta b_{jl}.$$

Of course, this implies that market clearing is possible. The division of bond holdings that individuals have from firms simply needs to be distributed accordingly. In general (as long as $\sum_{l=1}^{L} J_l > 1$), there will be infinitely many ways of achieving this. At least one way is definitely feasible, namely the above special case. We conclude that market clearing is uncritical. □

One objection that could still be made is that we have so far not commented on financial market completeness. Indeed, stock payoffs from firm (j,l) change in a fundamental way (that is, additively rather than multiplicatively) when increasing b_{jl}, namely by some multiple of the unit vector.[16] Notably, we have not assumed the entire state-space to be spanned, that is, merely the spanned subset of it must remain constant. This is almost always the case. The intuition for that lies within the above neutralization strategy: There exists an asset which can always compensate the change in the payoff structure of any stock, namely any bond (whose payoff is just that very unit vector). This implicitly assumes that bonds really exist in the discussed economies. That is, the above proof eliminates the two special cases $\boldsymbol{b} = \boldsymbol{0} \wedge \boldsymbol{\Delta b} \neq \boldsymbol{0}$ and $\boldsymbol{b} \neq \boldsymbol{0} \wedge \boldsymbol{\Delta b} = -\boldsymbol{b}$ for scenarios where the safe bond is a vital part of the marketed subspace. Generally, however, although stock payoffs change by construction (a point already noted by Rubinstein (2003), p. 11), the spanned subset of the state-space is unaltered.

4.2. General equilibrium MM: Bankruptcy in an arbitrary subset of states

While proving neutralizability in the simple case where there was no risk of bankruptcy was quite straightforward, things get cumbersome once we allow for more general debt structures. Here, rather than showing a concise strategy with which to negate the impact of additional debt issuance, we have to confine ourselves with showing "wealth neutrality" of changes Δb_{jl}. That is, we restrict ourselves to an analysis of the resulting change in total present value of consumption $p_0 c_{i0} + \sum_{s=1}^{S} r_s p_{ls} c_{ils}, i = 1,\dots,I$. Given a constant marketed subspace, each i can finance the previous $\boldsymbol{c}^*_i$ and nothing better as long as this present value does not change for her. Start again at an equilibrium:

Definition 2. $\left((\boldsymbol{c}^*_i, \boldsymbol{\theta}^*_i, \boldsymbol{a}^*_i, \boldsymbol{z}^*_i)_{i=1}^{I}, \boldsymbol{k}^*, \boldsymbol{p}^*, \boldsymbol{v}^*, \boldsymbol{R}^*, \boldsymbol{q}^*, \boldsymbol{b}\right)$ is an equilibrium of the stock-and-debt-economy with over-indebtedness (ESDO) if, given prices $\boldsymbol{p}^*, \boldsymbol{v}^*, \boldsymbol{R}^*, \boldsymbol{q}^*$ and bond supplies $\boldsymbol{b}$ such that $p_{ls} y_{jls} < b_{jl}$ for some $s = 1,\dots,S, j = 1,\dots,J_l, l = 1,\dots,L$,

[15] This last part proves that individuals indeed choose to neutralize the arisen consumption changes. Another way of seeing this is to note that nothing prevents us from choosing $\Delta b_{jl} < 0$ or, equivalently, to start at $\left((\boldsymbol{c}^*_i, \boldsymbol{\theta}^*_i, \boldsymbol{a}^*_i + \boldsymbol{\Delta a}^*_i, \boldsymbol{z}^*_i)_{i=1}^{I}, \boldsymbol{k}^*, \boldsymbol{p}^*, \boldsymbol{v}^*, \boldsymbol{R}^*, \boldsymbol{q}^*, \boldsymbol{b} + \boldsymbol{\Delta b}\right)$ and move on to $\left((\boldsymbol{c}^*_i, \boldsymbol{\theta}^*_i, \boldsymbol{a}^*_i, \boldsymbol{z}^*_i)_{i=1}^{I}, \boldsymbol{k}^*, \boldsymbol{p}^*, \boldsymbol{v}^*, \boldsymbol{R}^*, \boldsymbol{q}^*, \boldsymbol{b}\right)$, thereby showing the converse of what has been established.

[16] Of course, this additive change may become more complex once we allow for bankruptcy to happen.

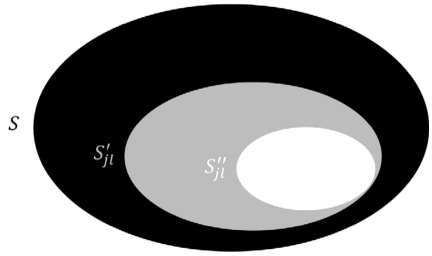

Fig. 1. Interrelations of the sets of states with solvency.

- Consumption $\boldsymbol{c}_i^*$ and portfolios $(\boldsymbol{\theta}_i^*, \boldsymbol{a}_i^*, \boldsymbol{z}_i^*)$ maximize consumption utility u_i subject to (1) and (2) for all $i = 1, \dots, I$.
- Capital choices $\boldsymbol{k}^*$ maximize SV for all $j = 1, \dots, J_l$, $l = 1, \dots, L$.
- Markets clear.

The definition of an ESDO generalizes the baseline equilibrium without over-indebtedness. However, it also demands that excessive debt is given for some firm in some state. With a constant marketed subspace, debt levels may be altered arbitrarily:

Theorem 2. *If $\left((\boldsymbol{c}_i^*, \boldsymbol{\theta}_i^*, \boldsymbol{a}_i^*, \boldsymbol{z}_i^*)_{i=1}^I, \boldsymbol{k}^*, \boldsymbol{p}^*, \boldsymbol{v}^*, \boldsymbol{R}^*, \boldsymbol{q}^*, \boldsymbol{b}\right)$ is an ESDO, then there exists some $\left((\boldsymbol{c}_i^*, \boldsymbol{\theta}_i^{**}, \boldsymbol{a}_i^{**}, \boldsymbol{z}_i^{**})_{i=1}^I, \boldsymbol{k}^*, \boldsymbol{p}^*, \boldsymbol{v}^{**}, \boldsymbol{R}^{**}, \boldsymbol{q}^*, \boldsymbol{b} + \boldsymbol{\Delta b}\right)$ that is also an ESDO as long as $\boldsymbol{b}$ and $\boldsymbol{b} + \boldsymbol{\Delta b}$ imply the same marketed subspace.*

Proof. Let $\boldsymbol{p}^*$, $\boldsymbol{r}^*$ and $\boldsymbol{q}^*$ be the same for both versions of the economy with over-indebtedness, and let $\boldsymbol{v}^{**}$ ensure constant SV. Optimum capital choices $\boldsymbol{k}^*$ are the same in both economies due to Lemma 1. Thus, state prices $\boldsymbol{r}^*$ are, in fact, also the same. Bond prices change from $\boldsymbol{R}^*$ to $\boldsymbol{R}^{**}$ due to the variation in debt levels, but result from these unaltered state prices. The change in firm values from $\boldsymbol{v}^*$ to $\boldsymbol{v}^{**}$ ensures constant shareholder values.

Note that we additionally assume unaltered spanning opportunities, that is, the marketed subspace has to stay the same. Otherwise, Rubinstein (2003, pp. 9–10) notes that state prices may change, thus reflecting not only the scarcity of consumption in certain states, but also the impossibility of some targeted consumption relocations.

To compare optimum levels of choice variables, we calculate differences in budget constraints, starting at $t = 0$ as taken from (10). A proper derivation is outsourced to Appendix A.2. We obtain

$$\sum_{l=1}^{L}\sum_{j=1}^{J_l}\left\{-\theta_{ijl}\left[\sum_{s\in S''_{jl}} r_s \Delta b_{jl} + \sum_{s\in S'_{jl}\setminus S''_{jl}} r_s(p_{ls}y_{jls} - b_{jl})\right] + \bar{\theta}_{ijl}\Delta(SV_{jl}) + a_{ijl}\left[-\sum_{s\in S'_{jl}\setminus S''_{jl}} r_s + \sum_{s\notin S''_{jl}} r_s \frac{p_{ls}y_{jls}}{b_{jl}+\Delta b_{jl}} - \sum_{s\notin S'_{jl}} r_s \frac{p_{ls}y_{jls}}{b_{jl}}\right]\right\}, \quad (15)$$

where we distinguish states without bankruptcy before debt has risen, S'_{jl}, and afterwards, S''_{jl}.[17] $\Delta(SV_{jl})$ refers to the change in shareholder value which can be taken from Lemma 2 to be zero. The remainder of (15) may be unequal to zero, plainly reflecting the fact that different levels of corporate debt force reallocations of consumption for those individuals that are in some way invested into this firm.[18] What remains to be established is how this reallocation is compensated. To do so, write down changes in the $t = 1$-BC as given by (2). They are

$$-\sum_{l=1}^{L}\sum_{j=1}^{J_l}\left\{\theta_{ijl}[\max(p_{ls}y_{jls} - b_{jl} - \Delta b_{jl}, 0) - \max(p_{ls}y_{jls} - b_{jl}, 0)] + \Delta\alpha_{ijls}\right\}, s = 1, \dots, S, \quad (16)$$

where the expressions for maximum operators and, correspondingly, their differences, as well as $\Delta\alpha_{ijls}$ depend on the exact kind of state we are looking at. A more detailed description of the following considerations is delegated to Appendix A.3.

It is sufficient to subdivide the set of all states into three proper subsets per firm.[19] Firstly, we begin with those states that display financial solvency of this (j, l) both before and after raising additional outside capital. Consider the two alternative solvency sets. We know that S'_{jl} is the set of states where (j, l) does not end up bankrupt with some level of debt b_{jl}. All the states in S''_{jl}, on the other hand, show solvency of this very firm with debt raised by $\Delta b_{jl} > 0$, which may be fewer in number and can never be more under our assumption of $\Delta b_{jl} > 0 \forall (j, l)$. Thus, we clearly obtain $S''_{jl} \subseteq S'_{jl}$. Fig. 1 illustrates this point.

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[17] If we wish to compare different versions of the economy in terms of outstanding debt of all firms, we have to be careful in notation. To avoid any potential confusion, firm-specific variables that refer to the case of increased debt will always be referred to using a double-prime indexation (i.e., we distinguish sets of solvency S''_{jl} versus S'_{jl}, but also, e.g., firm values v''_{jl} versus v_{jl}).

[18] Note that, since in an equilibrium all asset markets must clear, there is always at least one individual who is actually being influenced by the new debt level of (j, l) since the latter's shares and other assets must all belong to someone.

[19] The resulting subsets are always proper subsets of S given the firm goes bankrupt at least in some state given the lower debt level b_{jl}. Either of the following subsets may constitute the empty set.

The sets of solvency states before and after debt was raised can consequently be summarized as S''_{jl} and for them, the budget restriction (2) or, equivalently, the possible consumption expenditures $\sum_{l=1}^{L} p_{ls} c_{ils}$, change by

$$\sum_{l=1}^{L} \sum_{j=1}^{J_l} \theta_{ijl} \Delta b_{jl}.$$

Next, consider states for (j,l) where solvency was not even given before debt was raised. For these $s \in S \setminus S'_{jl}$ or, equivalently, $s \notin S'_{jl}$, (16) takes the form

$$-\sum_{l=1}^{L} \sum_{j=1}^{J_l} p_{ls} y_{jls} \left(\frac{a_{ijl}}{b_{jl} + \Delta b_{jl}} - \frac{a_{ijl}}{b_{jl}} \right).$$

Finally, we have to deal with states where the surge in debt was causal for over-indebtedness. This set is given by $S'_{jl} \setminus S''_{jl}$ and its concrete form of (16) is

$$-\sum_{l=1}^{L} \sum_{j=1}^{J_l} (-p_{ls} y_{jls} + b_{jl}) + \frac{a_{ijl} p_{ls} y_{jls}}{b_{jl} + \Delta b_{jl}} - a_{ijl}.$$

It can be readily seen that multiplying the above three types of changes by r_s and afterwards summing them over the corresponding sets of states per firm yields exactly the negative equivalent of (15). Hence, the net present value of possible new consumption expenditures is indeed equal to zero. Thus, no wealth effects arise, and as long as financial market completeness is not an issue, consumption profiles are permitted to remain the same once again.

It remains to show that markets clear in the $**$-economy. In fact, they do so by construction. To see this, start by noting that all of a firm's revenues must end up in some of its stakeholders' pockets, either stockholders' or bondholders'. This definitional relationship reads

$$p_{ls} y_{jls} = \max(p_{ls} y_{jls} - b_{jl} - \Delta b_{jl}, 0) \sum_{i=1}^{I} \theta_{ijl}^{**} + \min\left(\sum_{i=1}^{I} a_{ijl}^{**}, \frac{\sum_{i=1}^{I} a_{ijl}^{**}}{b_{jl} + \Delta b_{jl}} p_{ls} y_{jls} \right), \ j = 1, \dots, J_l, \ l = 1, \dots, L, \ s = 1, \dots, S, \tag{17}$$

where bond payoffs α_{ijls} are written using a minimum-operator as indicated by (3). Both the minimum and the maximum in (17) take on their respective first argument for $p_{ls} y_{jls} \geq b_{jl} + \Delta b_{jl}$. In the opposite case, (17) simplifies to

$$p_{ls} y_{jls} = \frac{\sum_{i=1}^{I} a_{ijl}^{**}}{b_{jl} + \Delta b_{jl}} p_{ls} y_{jls}.$$

The only way this can be true is for $\sum_{i=1}^{I} a_{ijl}^{**} = b_{jl} + \Delta b_{jl}$, i.e., if the bond market clears. Inserting this into (17) for states with no bankruptcy, i.e., those where $p_{ls} y_{jls} \geq b_{jl} + \Delta b_{jl}$ holds, gives rise to

$$p_{ls} y_{jls} = (p_{ls} y_{jls} - b_{jl} - \Delta b_{jl}) \sum_{i=1}^{I} \theta_{ijl}^{**} + b_{jl} + \Delta b_{jl},$$

which collapses to the stock market clearing condition $\sum_{i=1}^{I} \theta_{ijl}^{**} = 1$. So the stock market of each single firm must clear as well.

To recover security market clearing, one must start at the equilibrium of the $*$-economy. Take the $t = 1$-budget constraints from (2). In equilibrium, they hold with equality. That is, inserting the equilibrium values of all choice variables and summing over i, market clearing implies that

$$\sum_{l=1}^{L} p_{ls} \sum_{i=1}^{I} c_{ils}^{*} = \sum_{j=1}^{J_l} \sum_{l=1}^{L} p_{ls} y_{jls}, \ s = 1, \dots, S \tag{18}$$

holds true, i.e., that all production is consumed. Doing the same thing for the $**$-economy yields

$$\sum_{l=1}^{L} p_{ls} \sum_{i=1}^{I} c_{ils}^{**} = \sum_{j=1}^{J_l} \sum_{l=1}^{L} p_{ls} y_{jls} + \sum_{m=1}^{M} x_{ms} \sum_{i=1}^{I} z_{im}^{**}, \ s = 1, \dots, S \tag{19}$$

knowing that the markets for stocks and bonds clear, whereas securities market clearing is not yet superimposed. However, it has been shown that all individuals choose the same consumption profiles in both economies, $\boldsymbol{c}_i^{*} = \boldsymbol{c}_i^{**}$. As goods prices are also the same, the LHSs of (18) and (19) are identical. The only way that their RHSs can also be identical is then $\sum_{i=1}^{I} z_{im}^{**} = 0, m = 1, \dots, M$, i.e., securities market clearing. □

5. MM and SRI

What has been widely left out so far is the role of SRI in the context of varied debt. Solid conclusions are near impossible to reach here as we can assume virtually anything on how utility functions take account of the different valuations of assets they depend on. An intuitive way to proceed here would be to assume that i gains utility from the value of financial claims she holds towards any firm (j, l), but not from the number or kind of assets purchased from it. It then becomes apparent that the same claims towards all firms remain affordable without additional costs or benefits, such that a naïve neutralization strategy ensures neutrality of SRI.

Generally speaking, note that, provided that satiation in SRI holds for our reference equilibrium, individual utility improvements are impossible to be made. This is due to the fact that, while (j, l) can increase the number of bonds they issue, the same holds true for any individual via short sales. Since the latter do not compromise the number of solvency states S'_{jl} for that firm, they are the superior means of increasing bond supply and will in fact have already been performed to the optimum extent.

5.1. Neutrality of SRI with over-indebtedness

Carrying forth the logic from the final step in the proof of Theorem 2, we wish to relate an equilibrium with over-indebtedness and SRI (ESRIO) to an ESDO.

Definition 3. $\left((c_i^*, \theta_i^*, a_i^*, z_i^*)_{i=1}^I, \boldsymbol{k}^*, \boldsymbol{p}^*, \boldsymbol{v}^*, \boldsymbol{R}^*, \boldsymbol{q}^*, \boldsymbol{b}\right)$ is an equilibrium of the stock-and-debt-economy with socially responsible investing and over-indebtedness (ESRIO) if, given prices $\boldsymbol{p}^*, \boldsymbol{v}^*, \boldsymbol{R}^*, \boldsymbol{q}^*$ and bond supplies $\boldsymbol{b}$ such that $p_{ls} y_{jls} < b_{jl}$ for some $s = 1, \ldots, S, j = 1, \ldots, J_l, l = 1, \ldots, L$,

- Consumption $\boldsymbol{c}_i^*$ and portfolios $(\theta_i^*, \boldsymbol{a}_i^*, \boldsymbol{z}_i^*)$ maximize overall utility U_i subject to (1) and (2) for all $i = 1, \ldots, I$.
- Capital choices $\boldsymbol{k}^*$ maximize SV for all $j = 1, \ldots, J_l$, $l = 1, \ldots, L$.
- Markets clear.

Note that, in an equilibrium with SRI, the portfolio of individual i, $(\theta_i^*, \boldsymbol{a}_i^*, \boldsymbol{z}_i^*)$, is chosen such that all assets that grant utility are purchased to the point of satiation. The assumption of satiation in SRI may seem strong, but is shown to be reasonable through survey-based evidence by Dorfleitner and Nguyen (2016). All assets that grant disutility are avoided and the rest is used to adjust the consumption vector such that consumption utility is maximized.

Our goal is to show that every ESRIO is an ESDO and that for every ESDO, there exists an equivalent ESRIO with the same consumption choices by all individuals. This is the same step carried out by Arnold (2023), where an equilibrium with SRI (ESRI) is related to an ESDE.

Theorem 3. *Let $\left((\boldsymbol{c}_i^*, \theta_i^*, \boldsymbol{a}_i^*, \boldsymbol{z}_i^*)_{i=1}^I, \boldsymbol{k}^*, \boldsymbol{p}^*, \boldsymbol{v}^*, \boldsymbol{R}^*, \boldsymbol{q}^*, \boldsymbol{b}\right)$ be an ESRIO. Then it is also an ESDO.*

Proof. Start with an ESRIO as defined in Definition 3. Assume that, in an ESDO, $\boldsymbol{p}^*, \boldsymbol{v}^*, \boldsymbol{R}^*$ and $\boldsymbol{q}^*$ are the same as in that ESRIO. Let state prices $\boldsymbol{r}^*$ also be the same in both equilibria. Then the arg max of SV from (13), i.e., k_{jl}^* $\forall$ (j, l), is also identical. As a consequence, state prices as determined from (4) through (7) indeed have to be the same in the ESRIO and in the ESDO. All of this leads the budget constraints from (1) and (2) to remain identical. Thus, the same consumption vector $\boldsymbol{c}_i^*$ will be chosen. It can be achieved by the same portfolio $(\theta_i^*, \boldsymbol{a}_i^*, \boldsymbol{z}_i^*)$.[20] As all markets cleared in the ESRIO and neither supply nor demand of any good or asset increased or decreased, they continue to do so in the ESDO. Hence, the ESRIO $\left((\boldsymbol{c}_i^*, \theta_i^*, \boldsymbol{a}_i^*, \boldsymbol{z}_i^*)_{i=1}^I, \boldsymbol{k}^*, \boldsymbol{p}^*, \boldsymbol{v}^*, \boldsymbol{R}^*, \boldsymbol{q}^*, \boldsymbol{b}\right)$ is also an ESDO. $\square$

The converse of Theorem 3, that is, neutrality of the introduction of SRI when there is over-indebtedness, appears to be a more relevant result to show. To do so, an additional assumption about the marketed subspace is necessary. Concretely, SRI-concerns lead individuals to always wish to hold a certain number of stocks and bonds of firms they *care about*. Starting from this pre-determined sub-portfolio, only the rest of financial assets determine for each individual a personal marketed subspace. This subspace must for all individuals be identical to the one they had available before SRI entered their overall utility. For the special case of complete markets, this is just the *SWANS*-assumption. Again, moving away from the complete markets-requirement opens the possibility for additional cases, although they may seem unlikely.

Theorem 4. *Assume $\left((\boldsymbol{c}_i^*, \theta_i^*, \boldsymbol{a}_i^*, \boldsymbol{z}_i^*)_{i=1}^I, \boldsymbol{k}^*, \boldsymbol{p}^*, \boldsymbol{v}^*, \boldsymbol{R}^*, \boldsymbol{q}^*, \boldsymbol{b}\right)$ is an ESDO. Then, if no individual's personal marketed subspace is reduced when restricted to assets she is* indifferent towards*, there exists an ESRIO $\left((\boldsymbol{c}_i^*, \theta_i^{**}, \boldsymbol{a}_i^{**}, \boldsymbol{z}_i^{**})_{i=1}^I, \boldsymbol{k}^*, \boldsymbol{p}^*, \boldsymbol{v}^*, \boldsymbol{R}^*, \boldsymbol{q}^*, \boldsymbol{b}\right)$.*

Proof. As in the proof of Theorem 3, the assumption of constant $\boldsymbol{p}^*, \boldsymbol{v}^*, \boldsymbol{R}^*, \boldsymbol{q}^*$ and $\boldsymbol{r}^*$ yields identical production decisions and budget constraints.

It remains to show that the new portfolios $(\theta_i^{**}, \boldsymbol{a}_i^{**}, \boldsymbol{z}_i^{**})$ can and will indeed be used to finance $\boldsymbol{c}_i^*$ by all individuals. As it is assumed that individuals still face the same marketed subspace with SRI as they did without, these consumption vectors are indeed still affordable. Furthermore, no new consumption relocations are enabled. Thus, the same amount of wealth as before is available and will endogenously be distributed in the same proportions. $\square$

Theorems 3 and 4 taken together state that SRI, referring to both its abolishment and its introduction, respectively, is neutral. That is, SRI does not affect consumption or production choices in equilibrium. The upheld assumption of a constant marketed subspace using only assets individuals are indifferent towards for all individuals is a generalization of Arnold's (2023) *SWANS*-assumption:

[20] The mapping at this pont is one-to-many: there could be different portfolios than the one chosen with SRI considerations that enable consumption of $\boldsymbol{c}_i^*$.

$$\begin{tikzcd} \text{ESDE} \arrow[d, leftrightarrow] & \text{ESRI} \arrow[l] \\ \text{ESDO} & \text{ESRIO} \arrow[l] \end{tikzcd}$$

Fig. 2. Equilibrium relations.

the marketed subspace is required to stay constant either way, but here it need not have full dimension. The underlying assumption, however, remains strong. It implies that, for a given subset of assets an investor is *indifferent towards*, none of the other assets in the market are allowed to contribute to her spanning opportunities. This may point towards a violation of the model assumptions for the real world. However, the usefulness of socially responsible assets is then not motivated by the fact that they are ESG, but rather by the fact that they have unique payoff profiles.

In an ESDE, there are no risky bonds. That does not imply, however, that no other bond than the safe one contributes to market completeness in the ESDO-economy. In fact, it may well be the case that stocks, safe bonds and securities alone span the state-space no more after some firms over-indebt themselves.

So neutrality of SRI holds even in the presence of over-indebtedness. Note that, instead of searching for an ESRIO, we could have simply shown any $\left((\boldsymbol{c}_i^*, \theta_i', \boldsymbol{a}_i', \boldsymbol{z}_i')_{i=1}^I, \boldsymbol{k}^*, \boldsymbol{p}^*, \boldsymbol{v}^*, \boldsymbol{R}^*, \boldsymbol{q}^*, \boldsymbol{b}\right)$ to be another ESDO where we arbitrarily assign payoff-consistent portfolios to all i except one, where the latter is then tasked with guaranteeing market clearing and turns out to obtain the same payoffs as before, too (as is the case for the neutral i in Arnold's (2023) proof). So to every ESDO corresponds the basal one and, thus, also the ESRIO.

The logical chains relating equilibria are depicted in Fig. 2. It can be interpreted in the following way: An equilibrium from the arrow's starting point has an equilibrium with identical claims to all firms by all individuals at the arrow's endpoint. In the counterdirection, the figure indicates that an equilibrium with identical consumption vectors for each i exists, but portfolios possibly need to be shifted.

Notably, the allocation $(\boldsymbol{c}_i^*, \boldsymbol{k}^*)$ is the same across all equilibria, indicating irrelevance of both debt levels and SRI. So in the SRI-augmented Arrow–Debreu general equilibrium model, MM holds, SRI-neutrality holds and, consequently, the combination of both holds as well.

Fig. 2 also reveals that an ESRI is always a special case of an ESDE, and that an ESRIO is always a special case of an ESDO. Thus, linking the ESRI and the ESRIO to each other also necessitates that the restrictions on asset payoff structures implied by a constant marketed supspace from the respective proofs separately also hold simultaneously. This assumption is not innocuous. It says that if a stock or bond has become vital for the achieved marketed subspace when moving from an ESDE to an ESDO, every individual must be *indifferent towards* that asset. Else, SRI would proscribe a precise holding level of it for some individual which may run counter to the amount required to finance $\boldsymbol{c}_i^*$ in the ESRIO, but not in the ESRI. The converse holds for an asset that has become obsolete for the marketed subspace by new debt levels: it may prevent the establishment of $\boldsymbol{c}_i^*$ in an ESRI, but not in an ESRIO.[21]

5.2. Model limitations

Thus far, the model at hand suggests that, even in the presence of bankruptcy costs, corporate debt structures are irrelevant. On this basis, it can be shown that, given a constant marketed subspace, neither the introduction nor the abolishment of SRI have real economic consequences. However, it should be noted that strong assumptions have to be employed in order to reach this conclusion. It should thus be carefully interpreted as a benchmark.

Firstly, bankruptcy is "costless". That is, in case of a bankruptcy, no transaction costs arise. The entire firm proceeds are still available to bondholders. This may be disputed in the real world. For example, Branch (2002) argues that there are costs of dealing with financial distress. He provides an empirical value of around 16% of pre-distress value being lost in that way.

Secondly, trading involves no frictions. Information is perfectly symmetric between all actors. Taxes are absent. All of this leads to investors' capability to smoothly transfer wealth between time and states by paying state prices r_s. In reality, there may be asymmetries and frictions.

Thirdly, the assumption of a constant marketed subspace remains strong. As discussed, it encompasses complete markets as a special case, and should thus be viewed as a step in the right direction. However, it is still difficult to reconcile with reality.

Lastly, the assumptions made by Arnold (2023) have to carry over. These include the presence of a neutral investor and specific preferences that display both separability and satiation in SRI. This poses some limits on the permitted preferences of individuals.

Despite its limitations, the model provides some novel insights to the literature on general equilibrium, bankruptcy, and SRI. More specifically, it links the three topics and shows that, in an Arrow–Debreu general equilibrium model, SRI is neutral even in the presence of bankruptcy risks, provided the assumptions mentioned above hold.

[21] A plausible special case where even (dis-)liked bonds may contribute to the marketed subspace is if utilities depend on asset holdings only in relation to the financial claims implied therein, which was already noted at the beginning of this section.

6. Over-indebtedness as a means to enhance the marketed subspace

After what has been established, it is clear that the only channel through which over-indebtedness can potentially influence utility is by changing the marketed subspace. Consecutively, if financial markets were complete with some debt levels $\boldsymbol{b}$, no Pareto-improvement would be possible, even if debt levels differ. The converse argument (see also the discussion in Section 2), however, may also hold: given financial markets were incomplete under $\boldsymbol{b}$, a Pareto-improvement could be achieved by altered debt levels if that leads to financial market completeness.

The core idea is that the state-space spanned by given assets depends on the number of linearly independent payoff vectors. Some individuals may not have access to assets of a firm whose assets uniquely contribute to the spanned state-space. This is the case whenever they are not *indifferent towards* such a firm. They, thus, profit from over-indebtedness of another firm as long as the risky debt created in the process substitutes for the respective spanned subdimension. The results show some similarity to Nachman's (1989) idea of increasing the marketed subspace using options.

6.1. Conditions for usefulness of excessive debt

The model is simplified along the lines of Section 5 in Arnold (2023) for analytical convenience. Specifically, assume there is only one good, $L = 1$, such that firms need only be indexed by $j = 1, \dots, J$. Spot prices are unity, $p_0 = p_1 = 1$. Firm outputs obey multiplicative uncertainty:

$$f_{js}(k_j) = \lambda_{js} f_j(k_j), j = 1, \dots, J. \tag{20}$$

In (20), λ_{js} denotes a firm-specific shock to total factor productivity (TFP) in state $s \in S$.

For the simplified model, our first goal is to derive the necessary number of firms and states such that over-indebtedness may enhance the marketed subspace. Start with states. In the trivial case of $S = 1$, the financial market is always complete, provided non-zero output of some firm. For $S = 2$, a single firm with $\lambda_{j1} \neq \lambda_{j2}$, $\lambda_{js} > 0, s = 1, 2$, can ensure complete markets by issuing risk-free debt. Thus, the meaningfulness of risky debt requires at least $S = 3$ states.

It seems likely that $J = 2$ firms suffice to ensure a marketed subspace of full dimension. However, this is not generally the case. For instance, if $\lambda_{1s} = \lambda_{2s} = \lambda_s, s = 1, 2, 3$, i.e., if uncertainty hits the production side of the economy symmetrically, the financial market is destined to be incomplete without risky debt. Assuming symmetric TFP-shocks, revenue of firm j in state s is $\lambda_s f_j(k_j^*)$. Define firm 2's productivity multiplier as

$$m = \frac{f_2(k_2^*)}{f_1(k_1^*)}, \tag{21}$$

where $m \in (0, +\infty)$. We set $f_1(k_1^*)$ to unity for ease of exposition. That way, firm profit vectors, where the sth entry corresponds to the sth state, are $(\lambda_1, \lambda_2, \lambda_3)$ and $(m\lambda_1, m\lambda_2, m\lambda_3)$ for firm 1 and 2, respectively. Since each payoff vector is just a multiple of the respective other, the two firms' stocks span only one dimension of the three-dimensional state-space.

Now let firm 1 issue a safe bond. In order for it to be able to do so, we must assume $\lambda_s \geq 1, s = 1, 2, 3$.[22] The available payoff vectors are then

$$\begin{pmatrix} \lambda_1 - 1 \\ \lambda_2 - 1 \\ \lambda_3 - 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} m\lambda_1 \\ m\lambda_2 \\ m\lambda_3 \end{pmatrix}$$

for firm 1's stocks, firm 1's bond and firm 2's stocks, respectively. Intuitively, the bond emission has changed firm 1's stock payoffs in a way such that they are no longer multiples of firm 2's stock payoffs. Additionally, a new asset has become available. So the state-space may be spanned. However, one can show that the resulting payoff vectors are linearly dependent. As a consequence, the marketed subspace is not of full dimension. To see this, construct a portfolio with $\theta_{i1} = -1$ and $\theta_{i2} = 1/m$. The payoff from this portfolio is the unit vector, which is in turn just the safe bond's payoff vector. So a linear combination of two assets yields the third one's payoffs, rendering them collinear. In a similar vein, the state-space fails to be entirely spanned if firm 2 emerges as additional safe debt emittent. So those two firms cannot market the entire state-space if constrained to riskless debt.

The picture changes once we allow for risky debt. In order to be able to address in which states bankruptcy occurs, let them be ordered by profitability, $\lambda_1 < \lambda_2 < \lambda_3$. Let firm 1 issue $b_1 = \lambda_2$ units of risky bonds. Firm 2 issues a safe bond of volume $b_2 = 1$, where $m\lambda_s \geq 1, s = 1, 2, 3$, is superimposed. The available payoff vectors are then

$$\begin{pmatrix} 0 \\ 0 \\ \lambda_3 - \lambda_2 \end{pmatrix}, \begin{pmatrix} \lambda_1/\lambda_2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} m\lambda_1 - 1 \\ m\lambda_2 - 1 \\ m\lambda_3 - 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

for firm 1's stocks, firm 1's bond, firm 2's stocks and firm 2's bonds, respectively. Those four assets span the state-space iff we can construct an ADS for each state. This is possible. The ADS for $s = 1$ consists of a portfolio with $a_{i1} = -\lambda_2/(\lambda_2 - \lambda_1)$ and $a_{i2} = \lambda_2/(\lambda_2 - \lambda_1)$.

[22] Alternatively, one could let the firm issue a bond of some volume below unity. We stick with one entire bond for the sake of lucidity. Doing so involves no loss of generality.

To construct an ADS for $s = 2$, one needs to hold positions $\theta_{i1} = (m\lambda_3 - m\lambda_2 - 1)/(\lambda_3 - \lambda_2)$, $a_{i1} = (\lambda_2 + m\lambda_2^2 - m\lambda_1\lambda_2)/(\lambda_2 - \lambda_1)$, $\theta_{i2} = -1$ and $a_{i2} = m\lambda_2 - (m\lambda_2^2 + \lambda_2 - m\lambda_1\lambda_2)/(\lambda_2 - \lambda_1)$. Lastly, the ADS for $s = 3$ is trivially $\theta_{i1} = 1/(\lambda_3 - \lambda_2)$.

The above has shown that, without SRI, over-indebtedness can increase the dimension of the marketed subspace. To see inhowfar SRI is relevant, consider now the case of $S = 3$ states and $J = 3$ firms. Assume that the two first firms are symmetric save for the multiplicative difference as before. Without bankruptcy risk, only a two-dimensional space was marketed. Now let the third firm have stock payoffs such that the financial market becomes complete if all shares and the safe bond are accessible. However, if an individual i has SRI-preferences that proscribe her a certain holding level of firm 3's stocks (for example zero if she negatively screens that firm), she does not have access to a complete financial market given she wants to choose her optimum portfolio. Rather, she must come to a compromise between her desired consumption vector and portfolio. To restore a complete market for her, firm 1 must enter excessive debt as discussed above. Note that this implies that over-indebtedness can aid individuals only if they have no asset tastes for the relevant firm. However, the general result remains: In an economy with SRI, over-indebtedness of some firms may increase the marketed subspace of some individuals.

6.2. Increased subjective marketed subspace: example

Consider an economy where three firms ($J = 3$) produce the single $t = 1$-consumption good, $L = 1$, with spot price $p_1 = 1$. Their payoffs for the $S = 3$ states at $t = 1$ are $p_1 y_1 = p_1 y_2 = (3, 2, 1)$ and $p_1 y_3 = (1, 2, 1)$, where we stack states in vector notation as before. Firm 2 issues one bond, $b_2 = 1$, with payoff $(1, 1, 1)$. Consequently, the payoff on all stocks of firm 2 is $p_1 y_2 - b_2 = (2, 1, 0)$. For now, none of the other firms emit debt. Without SRI, the financial market is complete as the stocks of firm 1 together with the riskless bond and the stocks of firm 3 span the state-space. That is, one can construct ADSs for each state by

- $s = 1$: buying 50% of the shares of firm 1 while shorting those of firm 3 by the same amount,
- $s = 2$: buying 100% of the shares of firm 3 while shorting the bond and
- $s = 3$: buying 2 bonds while shorting firms 1 and 3 by 50% each.

When we introduce SRI as firm 3 being negatively screened by some i, however, the picture changes. *SWANS*-completeness of financial markets then necessitates the state-space to be spanned by assets of firms 1 and 2 alone (i.e., their respective shares and bond issuances). However, the synthesization of an $s = 2$-ADS is now simply impossible. The newly created difference between firms 1 and 2, which stems from firm 2 issuing a bond, did not create any newly available payment structures. If, on the other hand, firm 1 raises its bond issuance to $b_1 = 2$, there are now four different assets, three of which together allow i to construct that ADS. Investing directly into firm 1 delivers $p_1 y_1 - b_1 = (1, 0, 0)$ while its bond pays off $(1, 1, 0.5)$. So i simply has to buy the two bonds issued by firm 1 and sell both a 100%-share of firm 1 and the bond of firm 2 short. Equivalently, she could buy all firm-2-stocks and sell double of the firm-1-stocks short. As the ADS for $s = 1$ is just the firm-1-stock, and that for $s = 3$ can be obtained from two firm-2-bonds long with two firm-1-bonds short, the state-space is spanned. Hence, over-indebtedness of firm 1 has achieved financial market completeness in the stricter sense of *SWANS*, that is, with SRI.

7. Discussion and outlook

Frequently, it is claimed that the Modigliani-Miller theorem does not hold in the presence of bankruptcy risks. In a standard general equilibrium model, this is entirely due to the fact that the marketed subspace may change. The modeling framework of Arnold (2023) augments the general equilibrium model due to Arrow (1964) and Debreu (1959) in order to allow for an integration of Socially Responsible Investing into it. We show that, even in this generalized framework, allowing firms to go bankrupt does not have any real effects: shareholder value is unaltered and consumption possibilities remain the same as well.

What we need to employ in order to reach this stark conclusion is a per-individual notion of complete markets which must not be hampered via firms going bankrupt (at all instead of never or, generally speaking, more frequently). Notably, market completeness itself is actually not required as long as the kinds of market imperfections remain identical. Without over-indebtedness, this even is an automatism. Note, however, that the assumption of a constant marketed subspace, be it in the specific version of market completeness or with lower dimension, must be held up throughout the entire analysis.

The assumption that the marketed subspace is constant can easily be extended to include the presence of SRI uncritically. However, in that process, it becomes comparatively starker as SRI restricts the amount of assets that contribute to the spanned part of the state space for some individuals.

Our analysis comes with some drawbacks, the most salient of which is the maintained assumption of a constant marketed subspace. As Rubinstein (2003) argues, state prices may change in response to altered debt levels if this creates new securities or destroys existing ones, i.e., if the marketed subspace changes. If the latter happens, however, there may be benefits from over-indebtedness as it can, in fact, add marketed dimensions.

Another caveat is that we cannot make the distinction introduced by Green and Roth (2025) between *values-aligned* and *impact-aligned* investors, that is, people seeking to invest in firms based on whether these do good or can do more good with additional financing, respectively. The reason for this impossibility is that the general equilibrium structure together with the MM conclusion it carries leave no leeway for additional project financing via bonds. So even if there is *impact-alignment*, it cannot affect equilibrium.

We have assumed constant prices of both goods and states throughout the entire analysis. There are good reasons to believe in those two assumptions for the model as it stands, due to unaltered production quantities and marketed subspace, respectively. A caveat applies, however, once we take social preferences beyond portfolio construction into account: a responsible individual may not just shirk a disliked firm's assets, but also its products, thus possibly affecting prices via the goods demand channel. This idea is pursued by Hakenes and Schliephake (2026), who find that the combination of SRI with Socially Responsible Consumption leads to constant prices and considerable reductions of externalities.

It becomes apparent that SRI on its own can, at best, have limited effects on real economic activity. This implies that one cannot simply unload one's conscience onto the fact that the investments undertaken stem from responsible firms. It is either consumption or the factual act of making available new financial means to sound corporations that affect the environment, not trading.

Notation

i	Index for individuals, $i = 1, \dots, I$.
l	Index for goods, $l = 1, \dots, L$.
j	Good-specific index for firms, $j = 1, \dots, J_l$.
s	Index for states that can materialize at $t = 1$, $s = 1, \dots, S$.
u_i	Consumption utility of individual i according to $u_i(\boldsymbol{c_i}, \boldsymbol{k})$.
U_i	Overall utility of individual i according to $U_i(u_i(\boldsymbol{c_i}, \boldsymbol{k}), \boldsymbol{\theta_i}, \boldsymbol{a_i}, \boldsymbol{k}, \boldsymbol{v})$.
c_{i0}	Individual i's consumption of good 0.
p_0	Price of good 0 and physical capital.
c_{ils}	Individual i's consumption of good l in state s.
p_{ls}	Price of good l in state s.
$\bar{\theta}_{ijl}$	Individual i's initial share holdings of firm (j, l).
θ_{ijl}	Individual i's share holdings of firm (j, l) after trading.
v_{jl}	Firm value and stock price of firm (j, l).
k_{jl}	Physical capital input of firm (j, l).
y_{jls}	Productive output of firm (j, l) in state s according to $f_{jls}(k_{jl})$.
a_{ijl}	Individual i's bond holdings of firm (j, l).
α_{ijls}	Individual i's payoff from bond holdings of firm (j, l) in state s.
b_{jl}	Number of bonds issued by firm (j, l).
R_{jl}	Price of bonds issued by firm (j, l).
SV_{jl}	Shareholder value of firm (j, l) according to $v_{jl} - (p_0 k_{jl} - R_{jl} b_{jl})$.
S	Set of all possible states of nature.
S'_{jl}	Firm-specific set of states where firm (j, l) is not over-indebted given bond issuance b_{jl}, $p_{ls} y_{jls} \geq b_{jl}$, where $S'_{jl} \subseteq S$.
S''_{jl}	Firm-specific set of states where firm (j, l) is not over-indebted given bond issuance $b_{jl} + \Delta b_{jl}$, $p_{ls} y_{jls} \geq b_{jl} + \Delta b_{jl}$, where $S''_{jl} \subseteq S'_{jl}$.
z_{im}	Individual i's holdings of the mth zero-net supply asset.
q_m	Price of the mth zero-net supply asset.
r_s	Price of an asset or portfolio that delivers one unit of income in state s and nothing else.
R	Price of an asset or portfolio that delivers one unit of income in every state $s = 1, \dots, S$ according to $\sum_{s=1}^{S} r_s$.

CRediT authorship contribution statement

Fabian Alex: Writing – review & editing, Writing – original draft, Visualization, Validation, Investigation, Formal analysis, Conceptualization.

Declaration of competing interest

The author hereby assure that there are no conflicts of interest.

Appendix

The appendix serves to give comprehensive, rigorous proofs and derivations that would compromise readability of the main text if left there. More precisely, it takes a close look on how shareholder value SV_{jl} as well as the budget constraints (1) and (2) change when debt b_{jl} is varied by some small db_{jl} and a non-marginal amount Δb_{jl}, respectively.

A.1. Changes in SV

To begin with, allow firms to react to $db_{jl} > 0$ with $dk_{j,l} \neq 0$, leading to $dy_{jls} \neq 0$. That is, firms use the additionally gathered debt capital in order to adjust their business plan. In doing so, they change SV taken from (13) by

$$dSV_{jl} = \sum_{s=1}^{S} r_s p_{ls} dy_{jls} - p_0 dk_{jl},$$

which is equal to zero as long as

$$p_0 = \sum_{s=1}^{S} r_s p_{ls} \frac{dy_{jls}}{dk_{jl}}. \tag{A.1}$$

Since (A.1) is just (12), we can conclude that the corporation will optimally set $dk_{jl} = 0$ if it has already acted in an SV-maximizing way before debt was raised. Given the results of Section 3.1, this turns out to be a completely innocuous assumption. On the contrary, allowing $dk_{jl}/db_{jl} \neq 0$ would force us to assume an inefficient management given lower debt levels.

A.2. Changes in the BC for $t = 0$

The sources of change for the first period are threefold. Firstly, of course, everything being multiplied by b_{jl} changes by Δb_{jl}, but it does not stop there. Rather, two arguments of the BCs react to changes in bond issuance as well, one of them being firm values v_{jl} (but not shareholder values). They change by

$$\Delta v_{jl} = \sum_{s \in S''_{jl}} r_s (p_{ls} y_{jls} - b_{jl} - \Delta b_{jl}) - \sum_{s \in S'_{jl}} r_s (p_{ls} y_{jls} - b_{jl}).$$

Here, we can truncate some terms of $r_s(p_{ls} y_{jls} - b_{jl})$ because $S''_{jl} \subseteq S'_{jl}$. This leads to the expression

$$\Delta v_{jl} = -\sum_{s \in S''_{jl}} r_s \Delta b_{jl} - \sum_{s \in S'_{jl} \setminus S''_{jl}} r_s (p_{ls} y_{jls} - b_{jl}). \tag{A.2}$$

Furthermore, bond prices R_{jl} of the corresponding corporation (j, l) are also affected. Resulting from their definition (6), they experience a shift of magnitude

$$\Delta R_{jl} = \sum_{s \in S''_{jl}} r_s + \sum_{s \notin S''_{jl}} r_s \frac{p_{ls} y_{jls}}{b_{jl} + \Delta b_{jl}} - \sum_{s \in S'_{jl}} r_s - \sum_{s \notin S'_{jl}} r_s \frac{p_{ls} y_{jls}}{b_{jl}}$$

We can make use of basic set theory and omit some summands. More precisely, it is sums over r_s that leave only those addends from $S'_{jl} \setminus S''_{jl}$. We obtain

$$\Delta R_{jl} = -\sum_{s \in S'_{jl} \setminus S''_{jl}} r_s + \sum_{s \notin S''_{jl}} r_s \frac{p_{ls} y_{jls}}{b_{jl} + \Delta b_{jl}} - \sum_{s \notin S'_{jl}} r_s \frac{p_{ls} y_{jls}}{b_{jl}}. \tag{A.3}$$

Furthermore, what appears in the BC (1) is the product $R_{jl} b_{jl}$. Following (6), it changes by

$$\Delta(R_{jl} b_{jl}) = \sum_{s \in S''_{jl}} r_s (b_{jl} + \Delta b_{jl}) + \sum_{s \notin S''_{jl}} r_s p_{ls} y_{jls} - \sum_{s \in S'_{jl}} r_s b_{jl} - \sum_{s \notin S'_{jl}} r_s p_{ls} y_{jls},$$

which, analogously to the previous differences, simplifies to

$$\Delta(R_{jl} b_{jl}) = \sum_{s \in S''_{jl}} r_s \Delta b_{jl} - \sum_{s \in S'_{jl} \setminus S''_{jl}} r_s b_{jl}. \tag{A.4}$$

The absolute differences in parameters defined above will prove helpful in determining the change in wealth as implied by the first budget constraint.

To see how the BC for $t = 0$ changes, start with its formulation according to (10). Differencing it with and without raising debt by Δb_{jl} (assuming $dc_{i0} = 0$) results in

$$\sum_{l=1}^{L} \sum_{j=1}^{J_l} \left\{ -\bar{\theta}_{ijl} \left[\Delta v_{jl} + \Delta(R_{jl} b_{jl}) \right] + \Delta R_{jl} a_{ijl} \right\} + \sum_{l=1}^{L} \sum_{j=1}^{J_l} \theta_{ijl} \Delta v_{jl}.$$

Note that $\Delta v_{jl} + \Delta(R_{jl} b_{jl})$ is just $\Delta(SV_{jl})$, meaning that the term in square brackets reduces to zero. One can easily verify this using the definitions of differences from (A.2) and (A.4):

$$\Delta v_{jl} + \Delta(R_{jl} b_{jl}) = -\sum_{s \in S''_{jl}} r_s \Delta b_{jl} - \sum_{s \in S'_{jl} \setminus S''_{jl}} r_s (p_{ls} y_{jls} - b_{jl}) + \sum_{s \in S''_{jl}} r_s \Delta b_{jl} - \sum_{s \in S'_{jl} \setminus S''_{jl}} r_s b_{jl} = 0.$$

The remaining change after applying (A.2) and (A.3) is then

$$\sum_{l=1}^{L}\sum_{j=1}^{J_l}\left[-\sum_{s\in S''_{jl}} r_s\Delta b_{jl}-\sum_{s\in S'_{jl}\setminus S''_{jl}} r_s(p_{ls}y_{jls}-b_{jl})\right]a_{ijl}+\sum_{j=1}^{J_l}\theta_{ijl}\left(-\sum_{s\in S'_{jl}\setminus S''_{jl}} r_s+\sum_{s\notin S''_{jl}} r_s\frac{p_{ls}y_{jls}}{b_{jl}+\Delta b_{jl}}-\sum_{s\notin S'_{jl}} r_s\frac{p_{ls}y_{jls}}{b_{jl}}\right),$$

which is just (15). □

A.3. *Changes in the BC for* $t=1$

The approach undertaken to evaluate wealth changes in $t=1$ is somewhat different from the previous one. Here, we need to assess (16) for firm (j,l) depending on the state s that has materialized before weighting those changes with the adequate state price r_s each in order to make them directly comparable to the changes in $t=0$ taken from (15). To do so, start with the simplest subset of states: those where bankruptcy never occurs, neither with lower nor with higher debt levels. This set reads $S'_{jl}\cap S''_{jl}$, which simplifies to the smaller of both, namely S''_{jl}, due to the fact that $S''_{jl}\subseteq S'_{jl}$. For any such state, both maximum operators in (16) obtain their first element. Furthermore, α_{ijls} is just a_{ijl} for both debt levels, implying $d\alpha_{ijls}=0$. Thus, we get a total change in the budget constraint of

$$\sum_{l=1}^{L}\sum_{j=1}^{J_l}\theta_{ijl}\Delta b_{jl} \tag{A.5}$$

for all $s\in S''_{jl}$.

Next, look at states where bankruptcy was inevitable: Neither the increased nor the benchmark level of debt would have led to solvency. Formally, we have this case for all (j,l) such that $s\notin S'_{jl}$ and $s\notin S''_{jl}$. This intersection is simply the set of all $s\notin S'_{jl}$ because $S''_{jl}\subseteq S'_{jl}$. Here, both maximum operators obtain their second entry, namely 0, while $d\alpha_{ijls}\neq 0$ leads to a change

$$-\sum_{l=1}^{L}\sum_{j=1}^{J_l}p_{ls}y_{jls}\left(\frac{a_{ijl}}{b_{jl}+\Delta b_{jl}}-\frac{a_{ijl}}{b_{jl}}\right) \tag{A.6}$$

for any $s\notin S'_{jl}$.

Finally, consider those states where the additional debt is responsible for over-indebtedness. We intersect sets of solvency before debt was raised, $s\in S'_{jl}$, with sets of bankruptcy after the surge in debt occurred, $s\notin S''_{jl}$. This leads to $s\in S'_{jl}\setminus S''_{jl}$. With two different realizations of the maximum operator and α_{ijls}, we end up with

$$-\sum_{l=1}^{L}\sum_{j=1}^{J_l}(-p_{ls}y_{jls}+b_{jl})+\frac{a_{ijl}p_{ls}y_{jls}}{b_{jl}+\Delta b_{jl}}-a_{ijl} \tag{A.7}$$

for all $s\in S'_{jl}\setminus S''_{jl}$.

Before we can move on to add up all changes, i.e., (A.5), (A.6) and (A.7), make sure that each state is neither omitted nor counted twice in this procedure. Indeed, the intersection of $s\in S''_{jl}$ and $s\notin S'_{jl}$ is just the empty set $\emptyset$, as is that of $s\notin S'_{jl}$ with $s\in S'_{jl}\setminus S''_{jl}$. The same holds for $(S'_{jl}\setminus S''_{jl})\cap S''_{jl}$. The overall set union of those three, on the other hand, is veritably S. Thus, our partitioning of sets was indeed genuine.

To arrive at the MM-conclusion, weight all changes by corresponding state prices and add them up. As a consequence of the above calculations, we get a total $t=1$-originated wealth change of size

$$\sum_{s\in S''_{jl}} r_s\left(\sum_{l=1}^{L}\sum_{j=1}^{J_l}\theta_{ijl}\Delta b_{jl}\right)+\sum_{s\notin S'_{jl}} r_s\left[-\sum_{l=1}^{L}\sum_{j=1}^{J_l}p_{ls}y_{jls}\left(\frac{a_{ijl}}{b_{jl}+\Delta b_{jl}}-\frac{a_{ijl}}{b_{jl}}\right)\right]+$$
$$\sum_{s\in S'_{jl}\setminus S''_{jl}} r_s\left[-\sum_{l=1}^{L}\sum_{j=1}^{J_l}(-p_{ls}y_{jls}+b_{jl})+\frac{a_{ijl}p_{ls}y_{jls}}{b_{jl}+\Delta b_{jl}}-a_{ijl}\right]. \tag{A.8}$$

The final step in our proof is to add up (15) and (A.8). This yields a present value of wealth changes which is equal to zero. Thus, changing the debt structure of firms does not alter individual consumption possibilities for all $i=1,\dots,I$ (given the marketed subspace is unchanged). Hence, MM holds. □

Data availability

No data was used for the research described in the article.

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