DEUTSCHE
BUNDESBANK

Credit Risk Factor Modeling and the Basel II IRB Approach

Alfred Hamerle
(University of Regensburg)

Thilo Liebig
(Deutsche Bundesbank)

Daniel Rösch
(University of Regensburg)

Discussion Paper
Series 2: Banking and Financial Supervision
No 02/2003

Deutsche Bundesbank, Wilhelm-Epstein-Strasse 14, 60431 Frankfurt am Main,
Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 9566-1
Telex within Germany 41227, telex from abroad 414431, fax +49 69 5601071

Please address all orders in writing to: Deutsche Bundesbank,
Press and Public Relations Division, at the above address or via fax No +49 69 9566-3077

ISBN 3–935821–70–0

Credit Risk Factor Modeling and the Basel II IRB Approach*

Abstract

Default probabilities (PDs) and correlations play a crucial role in the New Basel Capital Accord. In commercial credit risk models they are an important constituent. Yet, modeling and estimation of PDs and correlations is still under active discussion. We show how the Basel II one factor model which is used to calibrate risk weights can be extended to a model for estimating PDs and correlations. The important advantage of this model is that it uses actual information about the point in time of the credit cycle. Thus, uncertainties about the parameters which are needed for Value-at-Risk calculations in portfolio models may be substantially reduced. First empirical evidence for the appropriateness of the models and underlying risk factors is given with S&P data.

Keywords: Credit Risk, Credit Ratings, Probability of Default, Bank Regulation
JEL Classification: C1, G21

* This paper was presented in October 2002 on the 9th annual meeting of the German Finance Association in Cologne. We thank the participants of the conference for helpful suggestions.

Zusammenfassung

Ausfallwahrscheinlichkeiten (PDs) und Korrelationen spielen eine entscheidende Rolle in den geplanten neuen Eigenmittelvereinbarungen (Basel II). Des Weiteren sind sie ein wichtiger Bestandteil von Kreditrisikomodellen. Dennoch existieren bei der Modellierung und Schätzung von PDs noch viele offene Fragen. Wir zeigen in diesem Papier wie das Ein-Faktor-Modell, das im Rahmen von Basel II zur Kalibrierung der Risikogewichte verwendet wird, erweitert werden kann, um die Schätzung von PDs und Korrelationen zu ermöglichen. Der entscheidende Vorteil dieses Modells besteht darin, dass es die jeweils aktuelle Information über den Stand des Konjunkturzykluses verwendet. Auf diese Weise können die Unsicherheiten über die Parameter, die für die Berechnung des Value-at-Risks in Portfoliomodellen benötigt werden, deutlich reduziert werden. Mit Hilfe von Daten von S&P wird die Angemessenheit des Modells und der zugrundeliegenden Riskofaktoren empirisch untersucht.

Schlagwörter: Kreditrisiko, Ratingverfahren, Ausfallwahrscheinlichkeit, Bank Regulierung
JEL Klassifizierung: C1, G21

Contents

1 Introduction

In the light of permanently growing default rates and diminishing margins the measurement and controlling of credit risk is essential – especially in the field of corporate banking. This is also stated by the Basel Committee on Banking Supervision who published a third consultative document in April 2003. The paper was followed by the European Commission's third consultative document in July 2003 which essentially adopted the Basel notions.

The Basel documents (1999a, 1999b, 2000, 2001, 2003) describe several determinants of default risk, i.e. the Default Probability (PD), the Loss Given Default, the Exposure At Default, the Maturity, and Correlations. Modeling of PDs and correlations is stated as a key factor for credit risk models since value-at-risk calculations are very sensitive due to changes in these parameters. In many credit risk models such as CreditMetrics, CreditRisk+ or CreditPortfolioView, default probabilities are essential input parameters. For outlines of these models see Gupton et al. (1997), Credit Suisse Financial Products (1997), and Wilson (1997a, 1997b). Approaches for unifying the different credit risk models are found in Gordy (1998), Koyluoglu/Hickman (1998), and Crouhy et al. (2000).

In the literature on credit risk several different approaches for modeling default probabilities are suggested. These approaches use different assumptions and information which is not always available for every firm. The asset value model due to Merton (1974) and Black/Scholes (1973)[1] assumes that the market value of total assets is observable in principle. Furthermore the capital structure is explicitly considered and default happens if the value of total assets is lower than the value of liabilities. In contrast reduced form models and other spread-related approaches derive relations between the credit spread of the credit risky bond and the default probability (e.g., Jarrow/Turnbull, 1995, Duffie/Singleton, 1997 and 1999, Madan/Unal, 1998, and Lando, 1998). An essential part of the theory of these models is the risk neutral valuation under the absence of arbitrage opportunities. Therefore the existence of market prices is indispensable for practical applications (see Jarrow/Turnbull, 2000).

---

[1] Extensions of the model can be found in Black/Cox (1976), Merton (1977), Geske (1977), Longstaff/Schwartz (1995), and Zhou (2001).

Another class of models, especially for mid-size corporate customers which do not offer market prices use rating procedures for the assessment of credit quality. For an outline of these models see Jarrow/Lando/Turnbull (1997) and Altman/Saunders (1998). According to the rating a segmentation in homogenous rating grades is employed. For each grade a default rate is determined and it is assumed that all borrowers within a grade exhibit equal default probabilities. The default probability is estimated by the default rate of each grade, sometimes after calculating historical averages.

A recent approach of a generalized framework for credit risk models and for default probabilities is due to Koyluoglu/Hickman (1998). These authors consider approaches which are used in current credit risk models, i.e. the “market factor model” from CreditMetrics, the “default rate model” from CreditPortfolioView and the “probability generating model” from CreditRisk+. In these approaches the variation of the default probability can be attributed to one or more “systemic” factors which are randomly drawn from the same distribution at each point in time.

Regarding default correlations mainly one of two directions is followed. The first one models default correlations between two borrowers directly by assuming a two-dimensional correlated Brownian motion for the returns on the values of the firms’ assets which trigger the defaults. The second direction, see e.g. CreditPortfolioView or CreditRisk+, models correlations by borrowers’ exposures to common risk factors. Given the realizations of the risk factors, asset returns and, thus, defaults are uncorrelated. The idea behind the Basel II model follows this direction by assuming the existence of one non-observable contemporaneous risk factor which is responsible for correlations.

The purpose of the present paper is to extend the Basel II framework to a model which directly relates observable risk factors to the Basel II model. Thus, default correlations and, furthermore, default probabilities can be directly attributed to the risk factors.

First, the outline of the model which underlies the Basel II Accord is summarized. It is shown that it can be extended to a general approach in a way that allows not only for changes of the *conditional PD* which is due to a common unobservable random factor under the assumption of a constant unconditional PD. Rather it also allows for changes of *unconditional PDs* due to observable risk factors. This extends the existing models to a dynamic setting. Koyluoglu/Hickman (1998), Lucas et al. (2000) and others describe model set-ups that are static in the sense that the risk factors which drive default rates or rating migrations are assumed i.i.d. Here we explicitly allow for time dependency of unconditional PDs in a multi-period model.

Second, it is shown that these observable risk factors can be identified by using a generalization of factor models for stock returns which are widely used in the financial area. This combination allows for modeling the systematic changes of the unconditional PDs and the random changes of the conditional PDs simultaneously.

Third, it is shown how these combined factor models can be employed even if stock returns are unobservable and rather only default data are observable. This is important especially for small and medium firms which do not have any marketable assets. Thus, the model is an example for how PDs can be estimated within the Basel II IRB framework. It is shown that an appropriate threshold model connecting the unobservable returns on a firm's assets with the observable default event leads to random effects logit or probit regression models which are extensions of the Basel II model.

Finally, consequences for portfolio modeling in the extended Basel II model are discussed. It is shown how knowledge about the relevant risk factors influences the parameters and reduces asset and default correlations. Thus, generating loss distributions can be simplified and become easier to handle. The model is demonstrated by analyzing empirical ratings data from 1982 to 1999.

The paper proceeds as follows. Section 2 defines the default event. In section 3 the Basel II model, common factor models for stock returns and a combination of them which is consistent with Basel II are discussed. Section 4 shows how these models are extended to the case when only default data are available. Section 5 discusses the implications for portfolio models and risk management and offers empirical evidence. Section 6 provides the summary and the conclusion.

2 Default Event, Asset Value, and Default Probability

In probabilistic terms the default event is random. Any attempt to quantifying credit risk has to determine the *probability* of the default event (PD) within a given future time period.[2]

Below the subscript t denotes the actual time and a single period is considered – e.g. one year. The two possible states at time t for borrower i, i=1,...,N, "default" and "non-default" are modeled by the indicator variable Y_{it}^{*} with

---

[2] In addition, other parameters, such as Exposure At Default (EAD) or Loss Given Default (LGD) are necessary. The primary focus of this paper is on PD and correlations and we treat the other parameters as known.

$$Y_{it}^{*} = \begin{cases} 1 & \text{borrower } i \text{ defaults in } t \\ 0 & \text{else} \end{cases}$$

Consider a continuous-state variable Y_{it} which may be interpreted as the natural logarithm of the value of the firm's assets. The default event is equivalent to the value of a firm's assets crossing a threshold c_{it} at t, i.e.

$$Y_{it} < c_{it} \quad \Leftrightarrow \quad Y_{it}^{*} = 1 \qquad (* \ 1).$$

The one-period default probabilities can be characterized by successively building up a threshold model. The model is formulated in a discrete time setting. The successive progression results from the fact that the process of the asset value only reaches time t if default has not taken place until time $t-1$. Else the firm is liquidated and the process of the asset value terminates. This conditional probability of the asset value falling below the threshold at time t, given survival until time $t-1$, is denoted by

$$\lambda_{it} = P\left(Y_{it} < c_{it} \middle| \, Y_{it-1} \geq c_{it-1}\right). \qquad (* \ 2)$$

Equivalently the „time to default" T_i could be treated as the relevant random variable. Here, T_i is an integer variable since only whole periods are counted. $T_i = t$ means that default happens at time t, the condition $T_i > t-1$ means that firm *i* did not default before t. The conditional default probability

$$\lambda_{it} = P\left(T_i = t \mid T_i > t-1\right) \qquad (* \ 3)$$

is also called a discrete-time hazard rate.[3]

[3] Usually hazard rates are analyzed in continuous-time settings, especially in biometrics and reliability theory. In the field of stochastic processes an extension is often found by stochastic intensity models, which play an important role in reduced-form-models (see, for example, Jarrow/Turnbull, 2000, or Lando, 1998). Our focus, however, is on the discrete-time setting with a larger time horizon, e.g. one year, which is generally used by bank practitioners (see Basel Committee on Banking Supervision, 2001 and 2003). As it is common in biometrics we adopt the term hazard rate for the discrete-time setting (see Kalbfleisch/Prentice, 1980, chap. 2.4).

While it always can be assumed that the default event is observable, the observability of the value of a firm's assets Y_{it} depends on the available data. If the asset value is observable, for example if market values of equity or credit scores are used as proxies, then the model is linear. Otherwise a nonlinear model is estimated which treats the asset value as a latent variable. In the following section the common framework for both cases is presented.

3 The Basel II Model and a Generalized Factor Model Representation

3.1 The Basel II Model for Asset Returns

As it is common in finance the asset value itself is not modeled. Rather a model is built for the changes, i.e. the returns, on the logarithm of a firm's assets. The return on borrower *i*'s assets at time *t* is represented by the normal distributed random variable

$$R_{it} = Y_{it} - y_{it-1} \quad \sim N\left(\mu_{it}, \sigma_i^2\right) \qquad (*\ 4)$$

(*i*=1,…,*N*; *t*=1,...,*T*), where $E\left(R_{it} \mid y_{it-1}\right) = \mu_{it}$ and $\mathrm{Var}\left(R_{it} \mid y_{it-1}\right) = \sigma_i^2$ are mean and standard deviation respectively. In the Basel II model it is assumed that deviations of asset returns from their means are due to a factor model with one systematic factor F_t which affects all returns simultaneously and *N* idiosyncratic factors U_{it} which affect each return separately. The exposure b to the common factor is equal for all firms within a given risk segment (for example, industry or size), i.e.[4]

$$R_{it} = \mu_{it} + b\, F_t + \varpi\, U_{it} \qquad (*\ 5)$$

where

$$F_t \sim N(0,1), \quad U_{it} \sim N(0,1)$$

[4] Note that in the Basel II model the subscript "*t*" is not taken into account. Rather, standardized returns are directly modeled. Thus, model (* 5) with time-dependent μ_{it} represents an extension which has important implications for credit risk modeling throughout the paper.

(i=1,…,N; t=1,...,T) are normally distributed with mean zero and standard deviation one. Idiosyncratic movements are assumed to be independent from the systematic factor and independent for different borrowers. All random variables are serially independent. Thus, within a risk segment variance, covariance and correlations between borrower i's and j's returns are given by

$$\sigma_i^2 = \mathrm{Var}(R_{it}) = b^2 + \varpi^2 = \sigma^2$$

$$\sigma_{ij} = \mathrm{Cov}(R_{it}, R_{jt}) = b^2 \quad i \neq j$$

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \, \sigma_j} = \left(\frac{b}{\sigma}\right)^2 \quad i \neq j \qquad (* \ 6)$$

(i,j=1,…,N). The one-period probability of default is called the unconditional PD in the notion of Basel II. It is the probability of the asset value falling below the threshold given the parameters of the process but without information about the realization of the common random factor. In the Basel II model it can be written as

$$\lambda_{it} = P(Y_{it} < c_{it}) = P(R_{it} < c_{it} - y_{it-1}) = P\left(\frac{R_{it} - \mu_{it}}{\sigma} < \frac{c_{it} - y_{it-1} - \mu_{it}}{\sigma}\right) \qquad (* \ 7)$$
$$= \Phi(\alpha_{it})$$

where $\alpha_{it} := \frac{c_{it} - y_{it-1} - \mu_{it}}{\sigma}$ and $\Phi(.)$ denotes the cumulative standard normal distribution function. In the context of Basel II in addition to this unconditional PD a conditional PD, given the realization of the random factor, is important. The conditional PD is

$$\lambda_{it}(f_t) = P\left(\frac{R_{it} - \mu_{it}}{\sigma} < \alpha_{it} \mid f_t\right) = \Phi\left(\frac{\alpha_{it} - \frac{b}{\sigma} f_t}{\sqrt{1 - \left(\frac{b}{\sigma}\right)^2}}\right)$$

$$= \Phi\left(\frac{\Phi^{-1}(\lambda_{it}) - \frac{b}{\sigma} f_t}{\sqrt{1 - \left(\frac{b}{\sigma}\right)^2}}\right) \qquad (* \ 8).$$

where $\Phi^{-1}(.)$ denotes the inverse cumulative standard normal distribution function. In the proposal as of January 2001 a (worst case) realization f_t which the common factor does fall short of with only a (small) probability of 0.5% is assumed. Since the factor is standard normally distributed the value for f_t is –2.5758. In addition, in the Basel Accord as of January 2001 it is assumed that for each borrower a proportion of 20% of the total variance is explained by the systematic factors, i.e. $(b/\sigma)^2$ is set to 0.2. Thus, the conditional PD is[5]

$$\lambda_{it}(-2.5758) = \Phi\left(\frac{\Phi^{-1}(\lambda_{it}) + \sqrt{0.2} \cdot 2.5758}{\sqrt{0.8}}\right)$$

In the Basel model it is assumed that the unconditional PDs do not vary over time. However, from (* 7) it is easily seen that λ_{it} varies through time if the mean asset returns μ_{it} are dynamic. Therefore, an extension of the Basel approach via a dynamic modeling of the unconditional and conditional PDs is necessary. The next section presents some common models and discusses how these input parameters can be estimated realistically and consistently with Basel II.

[5] In April 2003 the Basel Committee released a third consultative document. The essential differences are that the confidence level for the common factor has been increased to 99.9% and the exposure b to the factor is determined as a function of the unconditional PD and total annual sales of a corporate.

3.2 Factor Models for Stock Returns

The Basel II model only specifies the generating process for unexpected asset returns, i.e. the deviations of asset returns from their means. The generation of mean returns and, thus, for the unconditional PDs are left open. In many applications the generation of mean returns is explained by empirical factor models for stock returns. This section discusses the three types of factor models which are common in financial literature and practice. Each one imposes certain restrictions on the parameters due to the availability of time-series or cross-sectional data.

3.2.1 Statistical Factor Models

Statistical factor models use maximum-likelihood and principal-components-based factor analysis procedures to identify the factors which drive stock returns.[6] In general, K common factors may exist and the model is of the form

$$R_{it} = \mu_i + b_{i1} F_{1t} + \ldots + b_{iK} F_{Kt} + \varpi_i U_{it} \qquad (*\ 9)$$

where

$$F_{kt} \sim N(0,1), \qquad U_{it} \sim N(0,1)$$

(i=1,…,N; t=1,...,T, k=1,…,K) are normally distributed with mean zero and standard deviation one. Idiosyncratic movements are assumed to be independent from the systematic factors and independent for different borrowers. Moreover the common factors are assumed to be independent. To estimate factors and factor betas accurately a long and stable history of asset returns is required. All parameters must be constant over time, since every observation is treated as a random realization drawn from an identical distribution. Thus, on the one hand the model is more general than model (* 5) since K factors are allowed. On the other hand for the estimation of factors and factor exposures from time-series/cross-sectional data the stationarity assumption requires that $\mu_{it} = \mu_i$ for all t

6 See Connor (1995) for a survey.

(t=1,..., T). This also means that default probabilities are constant through time. A discussion and generalization of this will follow in section 3.3.

3.2.2 Macroeconomic or Index Factor Models

Macroeconomic models or index models use stock indices or observable economic time series as factors, such as inflation risk, GDP change or interest rates. In this case the regression model is of the form

$$R_{it} = \beta_{0i} + \boldsymbol{\beta}_i{}' \boldsymbol{Z}_t + \varpi_i U_{it} \qquad (* 10)$$

(i=1,…,N; t=1,...,T), where β_{0i} and $\boldsymbol{\beta}_i$ are unknown firm specific parameters to be estimated. The seminal reference for this kind of model is Chen/Roll/Ross (1986). For the estimation a time-series regression model is employed where the stationarity assumptions $E(\boldsymbol{Z}_t) = E(\boldsymbol{Z})$ and $\text{Cov}(\boldsymbol{Z}_t) = \text{Cov}(\boldsymbol{Z})$ for all t (t=1,..., T) must hold. In this case the expected asset returns $E(R_{it}) = \beta_{0i} + \boldsymbol{\beta}_i{}' E(\boldsymbol{Z})$ are time independent. Note also that expected returns in t conditional on realizations of the risk factors at $t-1$ $E(R_{it} | z_{t-1}) = \beta_{0i} + \boldsymbol{\beta}_i{}' E(\boldsymbol{Z})$ are also time-independent. Moreover, the asset returns are assumed to be serially independent.

This model is also found in the CreditMetrics framework where the risk factors are represented by contemporaneous stock index returns. The covariance between the standardized asset returns of firm i and j, resp. the correlation of asset returns is

$$\rho_{ij} = \frac{1}{\sigma_i \, \sigma_j} \boldsymbol{\beta}_i' \, \text{Cov}(\boldsymbol{Z}) \, \boldsymbol{\beta}_j$$

and is due to the exposures to the common observable risk factors or indices. For given realizations $\boldsymbol{Z}_t = \boldsymbol{z}_t$ of the risk factors the returns are independent.

3.2.3 Fundamental Factor Models

Fundamental factor models use firm specific information $\boldsymbol{X}_{it}$ as regressors, such as accounting data, firm size or index betas. The regression model is

$$R_{it} = \beta_{0t} + \boldsymbol{\gamma}_t{}' \boldsymbol{X}_{it} + \varpi_t U_{it} \qquad (* \; 11)$$

(i=1,…,N; t=1,...,T). $\boldsymbol{X}_{it}$ are interpreted as observable exposures to respective latent factors where $\boldsymbol{X}_{it} \sim N\left(E\left(\boldsymbol{X}_t\right), \mathrm{Cov}\left(\boldsymbol{X}_t\right)\right)$ (i=1,…,N; t=1,...,T). The vector $\boldsymbol{\gamma}_t$ are the expected returns of mimicking portfolios of the latent factors which are to be estimated by cross-sectional regression. The seminal references for this kind of models are Fama/MacBeth (1973) and Fama/French (1992). It is assumed that expected latent factor returns are constant through time, i.e. $\boldsymbol{\gamma}_t = \boldsymbol{\gamma}$ (t=1,...,T). Thus, the correlation between asset returns is

$$\rho_t = \frac{1}{\sigma_t \; \sigma_t} \boldsymbol{\gamma}' \; \mathrm{Cov}\left(\boldsymbol{X}_t\right) \boldsymbol{\gamma} \quad .$$

In this model expected asset returns are allowed to vary through time due to the variation of expected factor exposures $E\left(\boldsymbol{X}_t\right)$. For given realizations $\boldsymbol{X}_{it} = \boldsymbol{x}_{it}$ of the factor exposures the returns of two assets are independent. Recently these models are also found for explaining credit spread changes, see Duffee (1998), Pedrosa/Roll (1998) or Collin-Dufresne et al. (2001).

3.3 A Generalized Factor Model for Credit Risk

The empirical regression models from section 3.2 describe stock returns *either* as functions of macroeconomic time series *or* as functions of cross-sectional exposures. If time-series *and* cross-sectional data are available the models can be combined into one model which uses all information simultaneously. Furthermore this general model can be combined with the Basel II model where the existence of one common unobservable factor is assumed in addition. A model which combines fundamental, macroeconomic and statistical factor models is compatible with the Basel II model and it is able to explain the growth of firm values (and, thus, PDs) by various risk sources.

We assume that some risk factors affect the asset returns instantaneously and some risk factors affect the asset returns with a time lag. For example, some management decisions may cause a default immediately between $t-1$ and t while some past decisions have a lasting effect on default risk and their consequences are perceived at a later time. Note that the interpretation of the asset value is not limited to traded assets. Especially, small and medium firms do not have any traded assets. Markets which process information immediately into prices are not available. Therefore, lagged effects on the asset values do not contradict considerations about market efficiency.

Essentially the combined model is a *linear random effect panel model*. The notation which is used below refers to a particular risk segment (for example industry, or size group). It is assumed that the borrowers are homogenous within a risk segment regarding the relevant risk factors and the factor exposures. The parameters and the risk factors are allowed to differ between segments. Given the information until time *t* the model for a particular segment can be written as

$$R_{it} = \beta_{0t} + \boldsymbol{\gamma}_1{}'\boldsymbol{X}_{it} + \boldsymbol{\gamma}_2{}'\boldsymbol{x}_{it-1} + \boldsymbol{\beta}_1{}'\boldsymbol{Z}_t + \boldsymbol{\beta}_2{}'\boldsymbol{z}_{t-1} + b\,F_t + \varpi\,U_{it} \qquad (*\ 12)$$

(i=1,…,N; t=1,...,T), where $\boldsymbol{\beta}_{1i} = \boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_{2i} = \boldsymbol{\beta}_2$ are assumed for all borrowers within the risk segment. It follows that

$$E\left(R_{it} \mid \boldsymbol{x}_{it-1}, \boldsymbol{z}_{t-1}\right) = \beta_{0t} + \boldsymbol{\gamma}_1{}'E\left(\boldsymbol{X}_{it}\right) + \boldsymbol{\gamma}_2{}'\boldsymbol{x}_{it-1} + \boldsymbol{\beta}_1{}'E\left(\boldsymbol{Z}_t\right) + \boldsymbol{\beta}_2{}'\boldsymbol{z}_{t-1}.$$

$\boldsymbol{x}_{it-1}$ denotes a corresponding realization of a vector of lagged factors where the subscript $t-1$ represents arbitrary lags, i.e. also lags of two or more periods of time. z_{t-1} is a corresponding realization of a vector of lagged systematic risk factors. These realizations are known at time *t*. γ_1, γ_2, $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$ are suitably dimensioned parameter vectors.

For implementation in credit risk modelling proxies for the returns of the firms' assets are needed. If stock market data are available these returns can be approximated by stock returns analogously to CreditMetrics. However, many borrowers of a typical bank portfolio do not have any traded assets. Thus, other proxies for asset returns may be employed. The linear panel model can be established, for example with credit scores or credit score changes as in Hamerle/Rösch (2001). In general the distribution of the dependent variable at time *t* depends on the distribution of the risk factors. The goal of model (* 12) is to explain the dependent variable by lagged factors. Then all realizations of the risk factors are known at time t, the time-specific intercept vanishes and the distribution at time *t* is conditional on their lagged realizations. In this case the correlations are due to the random effect F_t. Furthermore it can be shown that the inclusion of lagged risk factors may supersede the random effect. Then conditional on all information at time t the dependent variables, and thus defaults, are independent. First empirical evidence for this model is given in Hamerle/Rösch (2001) where one lagged risk factor is able to explain much of the correlation in a sample of credit scores of German firms between 1989 and 1998.

Another possibility of estimation is to treat asset returns as latent variables which are not observable. Then, information must be gathered from observed default data. This leads to hazard-rate models which are discussed in the next section.

4 A Factor Model Representation for Default Data

In the latent model the realizations of the asset returns are not observable. Only the risk factors and the realization of the default indicator y_{it}^* are observable. Of interest is the PD in period t. The link between risk factors and PD is described by the threshold model (* 2). The model is usually formulated for given realizations of the risk factor and the factor exposures. For convenience, $\boldsymbol{x}_{it}$, $\boldsymbol{x}_{it-1}$, z_t and z_{t-1} are collected in the vector $\boldsymbol{w}_{it}$, and correspondingly the parameter vectors γ_1, γ_2, $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$ are collected in $\boldsymbol{\delta}$. Thus the linear term $\gamma_1'\boldsymbol{x}_{it}+\gamma_2'\boldsymbol{x}_{it-1}+\boldsymbol{\beta}_1'z_t+\boldsymbol{\beta}_2'z_{t-1}$ is abbreviated by $\boldsymbol{\delta}'\boldsymbol{w}_{it}$. Given that default has not happened before t one obtains for the conditional probability of default (CPD) (given information about the observable factors and factor exposures until time period $t-1$ and given the realization f_t of the random factor for time period t)

$$\begin{aligned}\lambda_{it}(\boldsymbol{w}_{it}, f_t) &= P\left(Y_{it}^* = 1 \mid \boldsymbol{w}_{it}, f_t\right) \\ &= P\left(U_{it} < \frac{c_{it} - y_{it-1} - \beta_{0t} - \boldsymbol{\delta}' \boldsymbol{w}_{it} - b\, f_t}{\varpi} \mid \boldsymbol{w}_{it}, f_t\right) \\ &= F\left(\tilde{\beta}_{0it} + \tilde{\boldsymbol{\delta}}' \boldsymbol{w}_{it} - \tilde{b}\, f_t \mid \boldsymbol{w}_{it}, f_t\right)\end{aligned} \tag{* 13}$$

where $\tilde{\beta}_{0it} := \frac{c_{it} - y_{it-1} - \beta_{0t}}{\varpi}$, $\tilde{\boldsymbol{\delta}} := -\frac{\boldsymbol{\delta}}{\varpi}$ and $\tilde{b} = \frac{b}{\varpi}$ represent reparameterizations of the regression parameters and F denotes the distribution function of the error term U_{it}. Neither the thresholds c_{it} nor the latent values y_{it-1} can be observed. So firm and time specific intercepts cannot be estimated. We restrict the intercept to β_{0t} and, for convenience, we use again the notation β_{0t}, b and $\boldsymbol{\delta}$ for the regression parameters.

Different assumptions about the error distribution function F lead to different models for the probability of default. The model underlying in Basel II is a random effects probit specification

$$\lambda_{it}(\boldsymbol{w}_{it}, f_t) = \Phi\left(\beta_{0t} + \boldsymbol{\delta}' \boldsymbol{w}_{it} - b\, f_t\right) \tag{* 14}$$

It is easily seen that (*14) is an extension of the Basel II model in (* 8) or (* 7) respectively. In (* 14) a generalized factor model for the expected return on a firm's assets is incorporated explaining the firm and time specific variations. A more tractable specification, which is widely used in applications of categorical regression models, is the logistic specification. The logistic model can be written as

$$\begin{aligned}\lambda_{it}(\boldsymbol{w}_{it}, f_t) &= \frac{\exp\left(\beta_{0t} + \boldsymbol{\delta}' \boldsymbol{w}_{it} - b\, f_t\right)}{1 + \exp\left(\beta_{0t} + \boldsymbol{\delta}' \boldsymbol{w}_{it} - b\, f_t\right)} \\ &= L\left(\beta_{0t} + \boldsymbol{\delta}' \boldsymbol{w}_{it} - b\, f_t\right)\end{aligned} \tag{* 15}$$

where $L(s) = \exp(s)/(1 + \exp(s))$. The "unconditional" probability of default (in Basel II terminology, meaning that we have only information about $\boldsymbol{w}_{it}$ but not about f_t) is given by

$$\lambda_{it}(\boldsymbol{w}_{it}) = \int_{-\infty}^{+\infty} F(\beta_{0t} + \boldsymbol{\delta}' \boldsymbol{w}_{it} - b\, f_t)\, \varphi(f_t)\, df_t \qquad (*\ 16)$$

where $\varphi(.)$ denotes the density function of the standard normal distribution. For example, for the logistic specification (* 15) one obtains

$$\lambda_{it}(\boldsymbol{w}_{it}) = \int_{-\infty}^{+\infty} L(\beta_{0t} + \boldsymbol{\delta}' \boldsymbol{w}_{it} - b\, f_t)\, \varphi(f_t)\, df_t \qquad (*\ 17).$$

In order to calculate the default correlations we consider again the linear random effect panel model (* 12) for the returns of a firm's assets. The linear term $\boldsymbol{\gamma}_1' \boldsymbol{x}_{it} + \boldsymbol{\gamma}_2' \boldsymbol{x}_{it-1} + \boldsymbol{\beta}_1' \boldsymbol{z}_t + \boldsymbol{\beta}_2' \boldsymbol{z}_{t-1}$ is again abbreviated by $\boldsymbol{\delta}' \boldsymbol{w}_{it}$. Then, for given values of $\boldsymbol{w}_{it}$, the asset correlation ρ, i.e. the correlation between R_{it} and R_{jt} $(i \neq j)$ within the risk segment is given by

$$\rho = \frac{b^2}{b^2 + \varpi^2}.$$

Moreover, for a given realization of the random factor f_t the returns R_{it} and R_{jt} are independent.

Since defaults are binary events the default correlations are determined by the probability

$$\lambda_{ijt}(\boldsymbol{w}_{it}, \boldsymbol{w}_{jt}) = P(Y_{it}^* = 1, Y_{jt}^* = 1 \mid \boldsymbol{w}_{it}, \boldsymbol{w}_{jt})$$

of joint defaults of borrowers *i* and *j*. Using (* 16) and the fact that for given f_t the defaults of borrowers *i* and *j* are independent, this joint probability is given by

$$\lambda_{ijt}(\boldsymbol{w}_{it}, \boldsymbol{w}_{jt}) = \int_{-\infty}^{+\infty} F(\beta_{0t} + \boldsymbol{\delta}' \boldsymbol{w}_{it} - b\, f_t)\, F(\beta_{0t} + \boldsymbol{\delta}' \boldsymbol{w}_{jt} - b\, f_t)\, \varphi(f_t)\, df_t \qquad (*\ 18).$$

Whereas the asset correlations between the returns for two firms within a risk segment are equal, this is no longer true for the corresponding default correlations which depend on the values of $\boldsymbol{w}_{it}$ and $\boldsymbol{w}_{jt}$ respectively. Furthermore, it should be noted that the variance

b^2 of the random effect plays a major role for the default correlation. If b^2 is not significantly different from zero, the random time effect may be neglected. Then, the default events within a risk segment may be considered (conditionally) independent.

5 Empirical Evidence and Implications for Risk Management and the Basel II IRB Approach

In this section empirical evidence about the generalized model is provided. Since long histories of real default data are scarce, we use ratings data from 1982 to 1999 from Standard & Poor's (2001b) which provide firm counts and defaults for seven rating grades from AAA to CCC.

For risk management and determining required capital in the context of Basle II, knowledge about the probability distribution of future losses, i.e. the losses which may arise in the next period $t+1$, is needed. Important input parameters for the loss distribution are the default probabilities λ_{it+1} and the default correlations $\rho\left(Y^*_{it+1}, Y^*_{jt+1}\right)$.

Firstly, we consider the Basel II model without explanatory risk factors. The model in Gordy/Heitfield (2000) for estimating asset correlations for rating grades is essentially a random effects probit model similar to (* 5). It is a one-factor model for the normalized i.i.d. returns $\tilde{R}_{it}$ on a firm's assets, i.e.

$$\tilde{R}_{it} = \sqrt{\rho}\, F_t + \sqrt{1-\rho}\, U_{it} \qquad (* \ 19)$$

where $\tilde{R}_{it} = \left(R_{it} - \mu_{it}\right)/\sigma_i$ and ρ as in (* 6). Again, it is assumed that a firm defaults if the return on its assets crosses a threshold. For each grade *g* it is assumed that there is a threshold α_{0g} so that $\Phi\left(\alpha_{0g}\right)$ is the unconditional PD of the grade. In comparison to (* 7) it is assumed that obligors within a grade are treated as homogeneous, and the restriction is imposed that each grade possesses an unconditional PD which does not depend on time. The model is estimated for the whole sample and for the particular rating grades.[7]

[7] Since the number of defaults in grades AAA to BBB is too small, the correlation parameter could not be estimated.

Table 1: Parameter estimates from model (19).*

	Parameter	Estimate	Standard Error	Pr > |t|
All Grades	SQRT(rho)	0.1978	0.03582	<.0001
	const.	-2.2490	0.05177	<.0001
BB	SQRT(rho)	0.2458	0.06908	0.0024
	const.	-2.2894	0.08119	<.0001
B	SQRT(rho)	0.2125	0.04358	0.0001
	const.	-1.6406	0.05870	<.0001
CCC	SQRT(rho)	0.2636	0.08082	0.0046
	const.	-0.8320	0.08512	<.0001

SQRT(rho) = $\sqrt{\rho}$

Table 1 shows the results for model (* 19). In the whole sample as well as in the risky rating grades the estimates for the parameter $\sqrt{\rho}$ are highly significantly different from zero. Asset correlations ρ are about 4% to 7%. Furthermore, the unconditional PD can be interpreted as the inverse of the standard normal distribution valued at the constant.

The next question is what happens if the restriction imposed does not hold and the PD is time dependent. Then, the model is misspecified. The main problem of this misspecification is that the expected returns on a firm's assets μ_{it} are falsely restricted to μ_i and the time-variation enters into the random effect. This will result in a systematic overestimation of the asset correlation.

Regarding correlations when the random effects probit or logit model (* 15) or (*16) are estimated, alternative scenarios may occur in general:

(1) The returns R_{it} can be fully explained by lagged factors and firm specific characteristics. For example we may have a rating score x_{it-1} established at the end of period $t-1$ that contains all information which is relevant for default risk in period *t*. Then all contemporaneous risk factors are insignificant. Moreover, the random effect is insignificant and can be neglected. In this case we can construct valid forecasts for the default probabilities λ_{it+1} at time t on the basis of the available information x_{it}, and the defaults are independent conditionally on this information. Let this case be called the "ideal case".

(2) Lagged factors, for example summarized in a rating score, are not sufficient to explain returns, and the random effect is significant. Then, contemporaneous factors

can be taken into the model, and their inclusion may render the random effect insignificant. In this case forecasts or stochastic models for the contemporaneous risk factors are needed. Alternatively one could explain as much as possible by lagged factors and keep the random effect in the model. In either case there is a further source of uncertainty which increases variances in comparison to case (1) and leads to correlations.

(3) As a modification of case (2) one puts the cyclicality into a time-specific intercept β_{0t}. Then the random effect should vanish. Here again, similar to case (2), one has the problem of forecasting β_{0t+1}.

(4) As a further modification one can include a rating score x_{it-1} and a lagged proxy for the macroeconomic environment, for example the overall default rates of time $t-1$ for explaining returns. If the intercept of the resulting model does not depend on time and the random effect is not significant one has essentially the ideal case (1).

In cases (1) and (4) the default correlation at $t+1$ is zero, conditional on the realizations of the risk factors at time *t*. Analogously the variances are reduced if information about the actual point of the credit cycle is used. This leads to significant simplifications and easier handling when loss distributions are generated.

To see this for the data on hand, model (* 19) is extended to model (* 12). Since we only have information about firm counts and defaults from S&P's, we cannot include firm specific information, such as size or financial ratios.[8] Macroeconomic data are provided by U.S. census, OECD, and Deutsche Bundesbank. Table 2 shows the results of model (* 12) for the whole sample as well as for the particular rating grades. In addition to model (* 19), contemporaneous risk factors and risk factors with a time-lag of one and two years are included.

8 Further empirical evidence for alternative models using individual ratings data can be found in Hamerle/Liebig/Scheule (2001).

Table 2: Parameter estimates from model (12) with macroeconomic data.*

	Parameter	Estimate	Standard Error	Pr > |t|
All Grades	SQRT(rho)	0.0967	0.02569	0.0017
	const.	-2.2145	0.03461	<.0001
	DSER_2	0.0980	0.03337	0.0097
	IND_IP	-0.1733	0.03923	0.0004
BB	SQRT(rho)	0.0821	0.08969	0.3728
	const.	-2.2529	0.05610	<.0001
	FEDR_1	0.2319	0.04779	0.0001
B	SQRT(rho)	0.0636	0.04410	0.1684
	const.	-1.6025	0.03848	<.0001
	UNEM_2	-0.0391	0.03655	0.3007
	DSER_2	0.0753	0.03935	0.0738
	IND_IP	-0.2155	0.04427	0.0002
CCC	SQRT(rho)	0.0724	0.1485	0.6321
	const.	-0.8569	0.05653	<0.0001
	UNEM_1	-0.2749	0.06705	0.0007

IND_IP: Change of Industrial Production
UNEM: Rate of Unemployment
FEDR: Federal Funds Rate
DSER: Percentage Change in Real Services Sector Value Added
_1: One-year time-lag
_2: Two-year time-lag
SQRT(rho) = $\sqrt{\rho}$

Firstly, from the first rows in Table 2 it can be seen that over all grades two risk factors are not enough to explain the random effect. We found that an additional variable does not lead to a further substantial decrease. However, the random effect variation can be reduced from 0.2 to approximately 0.1, leading to a reduction of asset correlation from 4% to approximately 1%. The reduction stems from one lagged variable and one contemporaneous variable (Change of Industrial Production). Thus, it seems that the whole sample of borrowers is too heterogeneous for being able to be fully explained by only few risk factors.

Then, the rating grades are analysed separately. Table 2 shows that we found a few risk factors which are able to explain the variation of the random effect. In each grade, the effect is reduced and no longer significantly different from zero. In grade B, altogether three factors are needed and one of them is contemporaneous (Change of Industrial Production). This is an example of case (2). For generating loss distributions this common factor has to be forecasted or simulated. Thus, in general correlations remain significant.

In grades BB and CCC, however, there is one single factor each, which explains the correlation. In addition, this factor drives the default risk with a time lag. These grades are examples for the ideal case (4) without additional variables needed. Defaults in these grades are independent, conditional on the value of the respective risk factor. In grade BB, an increase of the Federal Funds Rate comes along with an increase of default probabilities in the following year. This is reasonable because higher rates may lead to higher interest rates for debt. In grade CCC, higher unemployment is associated with decreasing default probabilities in the next year. This may also be plausible for two reasons. Firstly, if firms take measures for rationalizations they release employees. In the following years, their cost pressures decrease leading to lower default risk. Secondly, the state may stimulate the economy by higher public expenditures in times of higher unemployment. This could also decrease default risk.

Some comments are in order. The identified factors should be interpreted only as proxies for the underlying risk drivers. It is not meant to say, for example, that a higher Fund Rate is virtually responsible for default risk. Furthermore, we do not mean to have completely identified the whole risk structure. In banks' portfolios there may be different structures than in the portfolio underlying the S&P data. In addition, only one risk factor may not be enough for explaining correlations. Rather, a rating score or additional variables will have to be included. Nevertheless, we think that the shown results give first evidence for the appropriateness of a dynamic view of credit risk by model (* 12) and the empirical identification of systematic credit risk factors.

6 Summary and Conclusion

In the new Basel capital accord default probabilities and asset or default correlations are key factors for determining risk weights under the IRB approach. Modeling and estimation of default probabilities and default correlations are central for credit risk models since value-at-risk calculations are very sensitive due to changes of these parameters.

The model which is assumed in the Basle Capital Accord starts with given *unconditional PDs* and models conditional PDs for a special realization of an unknown risk factor. It does not prescribe how to model these unconditional PDs. The purpose of the present paper is to pick up the suggestion of the New Capital Accord and to show how *unconditional PDs* can be modelled within the Basel II framework. Thus, the Basel II model is extended to an explanatory model for the process which generates asset returns and default probability.

It is shown that the latent model for the returns on a firm's assets is essentially a linear random effects panel model. In the case when proxies for asset returns are observable, this linear model can be used. When only defaults are observable, an appropriate threshold model leads to a non-linear random effects probit or logit model. The important advantage of the model is that it uses actual information about the point in time of the credit cycle. By this, uncertainties in portfolio Value-at-Risk calculations may be substantially reduce. First empirical evidence for the appropriateness of these models and underlying risk factors is given with ratings data.

References

Altman, E.I, Saunders, A., 1998. Credit Risk Measurement: Developments Over the Last 20 Years. Journal of Banking and Finance 21, 1721-1742.

Bank of Italy, 2001. Calibration of Benchmark Risk Weights in the IRB Approach of Regulatory Capital Requirements, October.

Basel Committee on Banking Supervision, 2003. The New Basel Capital Accord. Consultative Document, April.Basel Committee on Banking Supervision, 2001. Potential Modifications to the Committee's Proposals, November.

Basel Committee on Banking Supervision, 2001. The New Basel Capital Accord. Consultative Document, January.

Basel Committee on Banking Supervision, 2000. Credit Ratings and Complementary Sources of Credit Quality Information. No 3, August.

Basel Committee on Banking Supervision, 1999b. A New Capital Adequacy Framework. No. 50, June.

Basel Committee on Banking Supervision, 1999a. Credit Risk Modeling: Current Practices and Applications. No. 49, April.

Black, F., Cox, J.C., 1976. Valuing Corporate Securities: Some Effects of Bond Indenture Provisions. Journal of Finance 31, 351-367.

Black, F., Scholes, M., 1973. The Pricing of Options and Corporate Liabilities. Journal of Political Economy, 637-654.

Chen, N.-F., Roll, R., Ross, S.A., 1986. Economic Forces and the Stock Market, Journal of Business 59, p. 383-403.

Collin-Dufresne, P., Goldstein, R.S., Martin, J.S., 2001. The Determinants of Credit Spread Changes. Journal of Finance, Vol. 56, December, 2135-2175.

Connor, G., 1995. The Three Types of Factor Models: A Comparison of Their Explanatory Power, Financial Analysts Journal, May-June, 42-46.

Credit Suisse Financial Products, 1997. CreditRisk+ - A Credit Risk Management Framework. London.

Crosbie, P., 1998. Modeling Default Risk. KMV Corporation, San Francisco.

Crouhy, M., Galai, D., Mark, R., 2001. Prototype Risk Rating System. Journal of Banking and Finance 25, 47-95.

Crouhy, M., Galai, D., Mark, R., 2000. A Comparative Analysis of Current Credit Risk Models. Journal of Banking and Finance 24, 59-117.

Duffee, G.R., 1998. The Relationship Between Treasury Yields and Corporate Bond Yield Spreads. Journal of Finance 53, 2225-2241.

Duffie, D., Singleton, K., 1999. Modeling Term Structures of Defaultable Bonds. Review of Financial Studies 12, 687-720.

Duffie, D., Singleton, K.J., 1997. An Econometric Model of the Term Structure of Interest-Rate Swap Yields. Journal of Finance 52, 1287-1322.

Efron, B., 1975. The Efficiency of Logistic Regression Compared to Normal Discriminant Analysis. Journal of the American Statistical Association 70, 892-898.

Fama, E.F., French, K.R., 1992. The Cross-Section of Expected Stock Returns. Journal of Finance 47, 427-465.

Finger, C.C., 2001. The One-Factor CreditMetrics Model in The New Basel Capital Accord. RiskMetrics Journal, Vol. 2(1), 9-18.

Finger, C.C., 1999. Conditional Approaches for CreditMetrics Portfolio Distributions. CreditMetrics Monitor, April, 14-33.

Geske, R., 1977. The Valuation of Corporate Liabilities as Compound Options. Journal of Financial and Quantitative Analysis 12, 541-552.

Gordy, M.B., 2001. A Risk-Factor Model Foundation for Ratings-Based Bank Capital Rules. Board of Governors of the Federal Reserve System. Working Paper.

Gordy, M.B., 2000. A Comparative Anatomy of Credit Risk Models. Journal of Banking and Finance 24, 119-149

Gordy, M.B., Heitfeld, N., 2000. Estimating Factor Loadings When Ratings Performance Data Are Scarce. Board of Governors of the Federal Reserve System, Division of Research and Statistics, Working Paper.

Gupton, G.M., Finger, C., Bhatia, M., 1997. Credit Metrics - Technical Document. Morgan Guaranty Trust Co., New York.

Hamerle, A., Liebig, T., Scheule, H., 2001. Dynamische Modellierung von Kreditrisiken mit zeitdiskreten Hazardraten. Working Paper, University of Regensburg, Deutsche Bundesbank.

Hamerle, A., Rösch, D., 2001. Risk Factors and Correlations for Credit Quality Changes. Forthcoming in: Zeitschrift für Betriebswirtschaftliche Forschung (in German).

Hamerle, A., Tutz, G., 1989. Diskrete Modelle zur Analyse von Verweildauern und Lebenszeiten. Frankfurt/Main, New York.

Jarrow, R.A., Turnbull, S.M., 2000. The Intersection of Market and Credit Risk. Journal of Banking and Finance 24, 271-299.

Jarrow, R.A., Lando,D., Turnbull, S.M., 1997. A Markov Model of the Term Structure of Credit Spreads. Review of Financial Studies 10.

Jarrow, R.A., Turnbull, S.M., 1995. Pricing Derivatives of Financial Securities Subject to Credit Risk. Journal of Finance 50, 53-85.

Kalbfleisch, J.D., Prentice, R.L., 1980. The Statistical Analysis of Failure Time Data. Wiley, New York.

Koyluoglu, H.U., Hickman, A., 1998. A Generalized Framework for Credit Risk Portfolio Models. Working Paper. Oliver, Wyman & Co. http://www.erisk.com/reference/archive/058_183GeneralizedFrameworkd.pdf

Lando, D., 1998. On Cox Processes and Credit Risky Securities. Review of Derivatives Research 2, 99-120.

Lo, A.W., 1985. Logit versus Discriminant Analysis – A Specification Test and Application to Corporate Bankruptcies. Journal of Econometrics 31, 151-178.

Longstaff, F.A., Schwartz, E.S., 1995. A Simple Approach to Valuing Risky Fixed and Floating Rate Debt. Journal of Finance 50, 789-819.

Lucas, A., Klaassen, P., Spreij, P., Straetmans, S., 2000. An Analytic Approach to Credit Risk of Large Corporate Bond and Loan Portfolios. Working Paper. Forthcoming in: Journal of Banking and Finance.

Madan, D., Unal, H., 1998. Pricing the Risks of Default. Review of Derivatives Research 2, 121-160.

Merton, R., 1977. On the Pricing of Contingent Claims and the Modigliani-Miller Theorem. Journal of Financial Economics 5, 241-249.

Merton, R., 1974. On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. Journal of Finance 29, 449-470.

Moody's Investor Service, 1998. Moody's Credit Ratings and Research. New York.

Ong, M.K., 1999. Internal Credit Risk Models – Capital Allocation and Performance Measurement. Risk Books, London.

Pedrosa, M., Roll, R., 2001. Systematic Risk in Corporate Bond Credit Spreads. Journal of Fixed Income 8, 7-26.

Standard & Poor's, (2001a). Corporate Ratings Criteria. New York.

Standard & Poor's (2001b) Ratings Performance 2000 – Default, Transition, Recovery, and Spreads. New York.

Tutz, G., 2000. Die Analyse kategorialer Daten – eine anwendungsorientierte Einführung in Logit-Modellierung und kategoriale Regression. München.

Wilson, T., 1997a. Portfolio Credit Risk I. Risk, September.

Wilson, T., 1997b. Portfolio Credit Risk II. Risk, October.

Zhou, C., 2001, An Analysis of Default Correlations and Multiple Defaults, Review of Financial Studies, Vol. 14, S. 555-576.