K U R T E. K L E I N

ASPECTS OF TRAFFIC-ZONING IN INVESTIGATING INNER-URBAN CONSUMER TRAFFIC WITH THE AID OF INTERACTION MODELS

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ZUSAMMENFASSUNG

Für einen Teil des innerstädtischen Einkaufsverkehrs gibt das Entropie-Maximierungsmodell von Wilson eine geeignete Erklärungsbasis ab. Die Verkehrsverteilungsmatrix wird durch die übertragbare Präferenzmatrix ersetzt: an ihr kann beispielhaft der Einfluß des Punktmusters auf die Modellergebnisse gezeigt werden. Um diesen zu minimieren, sind regelmäßige Punktmuster mit geringer Variation der Aufkommensgrößen Versand/Empfang gesucht. Ausgehend vom Hexagonalnetz läßt sich eine Transformation angeben, welche die Variation der Bevölkerungsdichte im innerstädtischen Bereich in den auszuwählenden Verkehrszellen minimiert. Regensburg dient als Anwendungsbeispiel für die vorgetragenen Überlegungen.

1. INTRODUCTION

For the succeeding considerations it is supposed that number and shape of traffic zones are to be determinated without restriction. In case of optimizing some basic pattern of traffic zones the works of Openshaw [14,15] could be referred.

Two main thoughts determine the following investigation:

(i) traffic zoning as regionalisation should square with the basic trip distribution hypothesis;

(ii) transfer of the results of model application should be granted; to put it in other words: minimizing the impact of point pattern which is formed by the centres of traffic zones and taken as a basis for model calibration.

ad(i). As there is no universal regionalisation for the purpose of traffic investigations important hints of how to choose and shape traffic zones can be drawn by careful investigation of the underlying trip distribution hypothesis or interaction model. As to those models with explicitly expressed hypothesis (e.g. route/time minimization) solutions like Thiessen-polygons constructed around sinks are easily offered. Difficulties are growing if one has to use models with implicitly given hypothesis like entropy-maximizing methods.

ad (ii). Close relations of trip distribution hypothesis with regionalisation are also desirable to minimize the - in its amount contested - impact of point pattern on the results of model application. At least three ways could be followed to filter out the influence of point pattern:

- construct a reference trip distribution without using spatial variables
- compare model results with those being derived by use of an ideal point pattern
- compute two trip distribution matrices, one with a global, the second with spatially differenciated impedance function(s).

Taking up the first two thoughts, section 2 deals with a suggestion how to construct a trip distribution without relying on areal variables. Results are used to derive some general properties of subdivision of space if the entropy-maximizing hypothesis is thought of being suited best for the problem under investigation. Section 3 focuses the general results of section 2 on the regionalisation problem of the multinodal inner-urban space, while section 4 contains an application of the ideas put forward.

Before going further some remarks should be made upon inner-urban consumer traffic. In view of consumer-behaviour it can be subdivided into at least two subgroups (SG):

SG1 : Time/Distance/Effort Minimization in satisfying high-frequency demand (mainly food). General interaction-theory offers 'Linear Optimization Models' in connection with the 'Theory of Central Places' and its shaping of market areas.

SG2 : With decreasing frequency of demand of goods traffic considerations take into account 'distance' in various measures, 'relative location' in form of accessibility or intervening opportunity and size/facilities/multiple choice of possible destinations. Apart from special behavioural hypothesis and dependent on available information intervening opportunity or entropy-maximizing models could be used.

In case of poor and rather global information - that is: application of entropy-maximizing models - the weight of underlying point pattern in analysing consumer traffic increases: explanation of trip distribution is based solely on spatial variables like 'distance', 'relative location' or derived measures like accessibility', 'potential' or 'areal competition'. The representation of the investigation area within the model especially the zoning completely determines the success of further interpretation. Therefore besides questions of measuring distance (real, ordinal, perceptive) or effort, expenditure etc. one has to raise the question about a neutral point pattern or zoning-system, too. Regular point patterns are well suited because they inherit the lowest information, overall as well as regionally differentiated.

2. IMPACT OF POINT PATTERN ON MODEL RESULTS

Interaction models generate a trip distribution matrix under use of

- information of local structure at sources/sinks
- characteristics of the investigation area (distance matrix, route network, barries, etc.)
- global indicator about traffic (total expenditure, etc)
- the trip distribution hypothesis.

Since these characteristics are dependent on space it is suggested to substitute the preference matrix G for the computed matrix.

Given:

$V: = \{V_1,\dots,V_n\}$ Set of traffic amount generated by origin zones $\{1,..,n\}$

(1) $E: = \{E_1,\dots,E_n\}$ ... destination zones $\{1,..,n\}$

$T: = \sum_{j=1}^{n} E_j$ Total amount of traffic

$= \sum_{i=1}^{n} V_i$

The matrix

(2) $$F = (F_{ij}) = \left(\frac{V_i \ E_j}{T}\right)$$

has the following properties:

- up to a multiplicative constant $\left(\frac{1}{T}\right)$ this is the matrix of the interaction probabilities under assumption that V and E are independent in sense of probability theory
- under given row and column sums this is the matrix of maximal entropy
- the following constraints are satisfied:

(3) $$\sum_{i=1}^{n} \frac{V_i \ E_j}{T} = E_j \quad \forall j \in \{1,..,n\}$$

(4) $$\sum_{j=1}^{n} \frac{V_i E_j}{T} = V_i \quad \forall_i \in \{1,...,n\}$$

The accompanying preference matrix $G=(G_{ij})$ of a real (or computed) trip distribution matrix $T= (T_{ij})$ will be defined as

(5) $$(G_{ij}) = \left(\frac{T_{ij}}{F_{ij}}\right)$$

Applied to the entropy-maximizing model one gets

(6) $$G_{ij} = \frac{A_i B_j V_i E_j d_{ij}^{-\alpha}}{\frac{V_i E_j}{T}} = \frac{1}{T} \ A_i B_j d_{ij}^{-\alpha},$$

($d_{ij}^{-\alpha}$: logarithmic perception of distance d between orig. i and dest.j)

whereas the balancing factors A, B are defined like

(7) $$A_i: = \frac{1}{\sum_{j=1}^{n} B_j E_j d_{ij}^{-\alpha}}, \quad B_j := \frac{1}{\sum_{i=1}^{n} A_i V_i d_{ij}^{-\alpha}}$$

As to the above described Wilson model the impact of point pattern on the computed trip distribution could be estimated by examining the balancing factors A, B. Since latter are computed with the aid of iterative methods [7] the initial step of the Furness procedure can serve as approximation. So we get for example:

(8) $$B_j \approx \frac{1}{\sum_{i=1}^{n} V_i d_{ij}^{-\alpha}}$$

A Taylor-expansion of the denominator holds:

(9) $$\sum_{i=1}^{n} V_i d_{ij}^{-\alpha} \approx n \left\{ \bar{V} \ d_{.j}^{-\alpha} + (-\alpha) d_{.j}^{-\alpha-1} \cdot \right.$$
$$\cdot \operatorname{cov}(V, d_{.j}) + \frac{1}{2} \bar{V} \ \sigma^2(d_{.j}) \cdot$$
$$\left. \cdot (-\alpha)(-\alpha-1) d_{.j}^{-\alpha-2} \right\}$$

and

(10) $$\operatorname{cov}(V, d_{.j}) := \frac{1}{n} \sum_{i=1}^{n} (V_i - \bar{V}) \ (d_{ij} - d_{.j})$$

$d_{.j}$:= average distance of trips/traffic to destination zone j.

Hence follows that the areal competition of senders at sink j : $\sum_{i=1}^{n} V_i d_j^{-\alpha}$ can be expressed in a first approximation as the product of average amount of traffic sent by sources and α-averaged distance of traffic with zone j as destination. From that can be concluded that the areal variation of B-values is dependent on to what extent the distribution of receiver-distances differs from sink to sink. Therefore pattern with homogeneous point-density will minimize this areal variation.

It still remains to discuss the second term of the Taylor expansion (10). While the first approximation is only regionalized by the relative location of destination zone j (expressed by $d_{\cdot j}^{-\alpha}$), its corrector term $\text{cov}(V, d_{\cdot j})$ takes into account the co-variation of amount of traffic at sources, V_i, and distance to sink j. Examples for the relation of point pattern and cov. cf. Klein [11, pp.191-196]. This by no means ignorable contribution to the areal competition can be minimized by

- probability theoretical independence of V and $d_{\cdot j}$
- or low areal variation of V.

At the point of entropy-maximizing method that much can be said that the impact of point pattern on model results can be delimited by

- use of a regular pattern as well as
- minimizing the areal variation of V and E.

3. DERIVATION OF A PATTERN TRANSFORMATION

This section deals with a transformation how a regular pattern can be adjusted to a special problem under consideration of the ideas developed so far.
The investigation area is overlaid with a hexagonal network. Latter seems to be particularly suited for traffic investigation within the multinodal inner-urban region. The centres of hexagons form also the point pattern required for model application. Within the hexagons persons act as potential tripmakers. Their areal distribution is shown in Fig. 1. It is evident that the number of persons in traffic zones (hexagons) varies considerably.

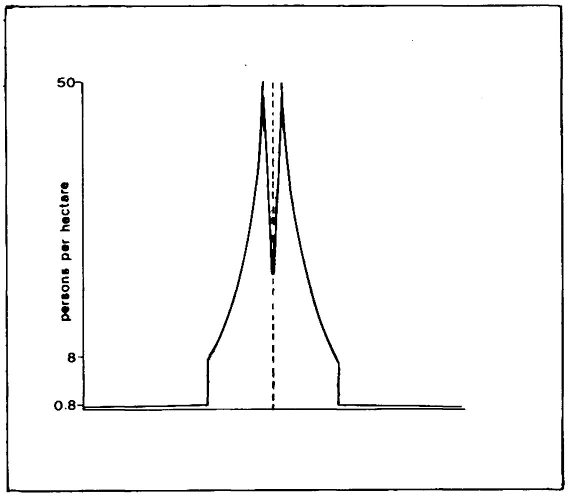

Fig. 1. Theoretical distribution of population density across a city centre. (Adapted from Dixon [6, p. 20])

So traffic zones are to be transformed to minimize this areal variation. Choose a transformation centre (x_m, y_m). Let $r_1, .., r_k$ denote the distances from (x_m, y_m) to those hexagon centres which will be touched by the half-profile shown in Fig. 1. F be the area of a hexagon.

$$(11)\quad V_l = \int_{F(r_l)} p(F)\,dF \qquad \forall l \in \{1, \ldots, k\}$$

p: function of probability density, shown in Fig. 1.

If $r_1 < r_2 < .. < r_k$ then - outside the city centre -

$V_1 > V_2 > \ .. > V_k$ and the decline will correspond to the one of p.

Transformation t should be chosen to achieve

$$\{r_1, \ldots, r_k\} \rightarrow \{r_1^{(t)}, \ldots, r_k^{(t)}\} \quad \text{with}$$

$$(12)\quad \bar{V} = V_l = \int_{F(r_l^{(t)})} p(F)\,dF \qquad \forall l \in \{1, .., k\}$$

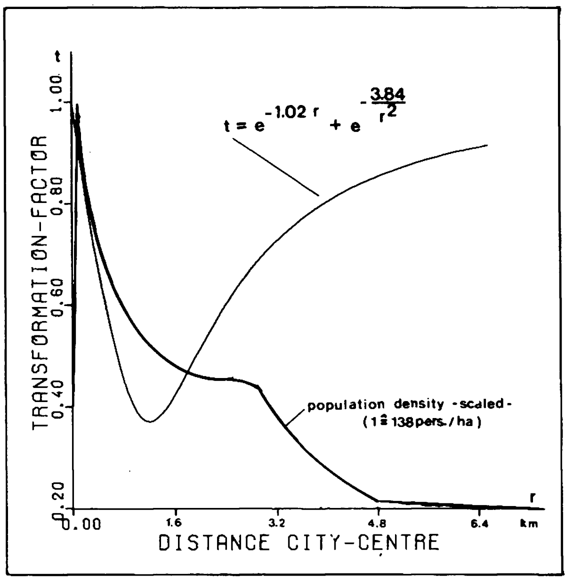

Fig. 2. Graph of transformation function for the case of neglecting barriers (cf. Fig. 4). Empirically derived graph of population density for Regensburg has been opposed.

To characterize the transformation function in words:

- expansion in the range of low population density
- contraction in the range of high

In the present case the following transformation function has been used:

$$(13)\quad t = e^{-ar} + e^{-\frac{b}{r^2}} \quad \text{(cf. Fig. 2)}$$

$r := d((x,y),\ (x_m,y_m))$

(x,y) : coordinates of a hexagon centre

$d(\ ,\ )$: distance to transformation centre (x_m,y_m)

$0 < a,b$ a : dependent on the extension of the CBD and the mesh-width of hexagonal network

b : dependent on the total urban and especially densely populated area (a first guess could be the square of the standard distance of population)

Each hexagon centre (x,y) will be transformed into $(x^{(t)},\ y^{(t)})$ like:

$$x^{(t)} = (x - x_m)\cdot t_{(x,y)} + x_m$$

$$(14)\quad y^{(t)} = (y - y_m)\cdot t_{(x,y)} + y_m$$

The numerical estimation of the transformation parameters a, b uses the object function :

$$(15)\quad ZF = \frac{1}{n}\sum_{i=1}^{n}(V_i - \bar{V})^2 = \min .$$

4. APPLICATION

Handling the information of a two years period study of inner-urban consumer traffic at Regensburg a zoning system for analysing SG 2(cf. 1.) is needed. With the method outlined above a first design will be worked out which has to be modified in small scale to take into consideration road network, distribution of shops and interaction barriers.

As in section 3 the areal distribution of sources will be thought as coincident with population distribution. As a consequence of its historical development and physical (river Danube, relief) as well as anthropogeneous (railway, highway) barriers the graph of population density differs from the ideal one (cf. Fig. 2.). There is no exponential decline from centre to fringe and the density gap of the CBD is only slightly marked.

As to possible sinks on the basis of a total survey about 70 inner-urban clusters of shops or shopping centres of differing size and facilities have been established. Only a small part of these is chosen as direct destination under the aspect of SG 2. Latter are located within the inner city, along arterial roads - mainly in the area of pre-1914 expansion and incorporated village centres - and at the urban fringe. However inquiries on consumer behaviour show that multi-purpose shopping trips occur frequently. So the whole 70 centres can be regarded as indirect destinations of SG2.

Regarding the shopping behaviour there is much evidence that the entropy-maximizing method with its interpretation of areal competition will be appropriate to reconstruct it. For there are not only centripetal traffic flows but also flows along the margin and CBD-crossing flows. Latter is due to the described pattern of sinks and the tendency of retail decentralisation.

A hexagonal zoning system of 69 cells

has been chosen which covers the whole settled area of Regensburg. With regard to population density Regensburg seems to be partly overbounded (cf. Fig. 3). Two calibration runs have been made which differ in the consideration of barriers. Results may be seen from Table 1.

Tab. 1 Calibration results of transformation parameters

	Without barr.		With barr.	
	Hexag.	Transf.	Hexag.	Transf.
$\bar{\bar{V}}$	2029	2029	2029	2029
ZF	2265	1738	2458	1858
V_{min}	0	0	0	0
V_{max}	7871	7593	10121	7541
$a\,[km^{-1}]$	-	1.02	-	1.02
$b\,[km^{2}]$	-	3.84	-	3.20

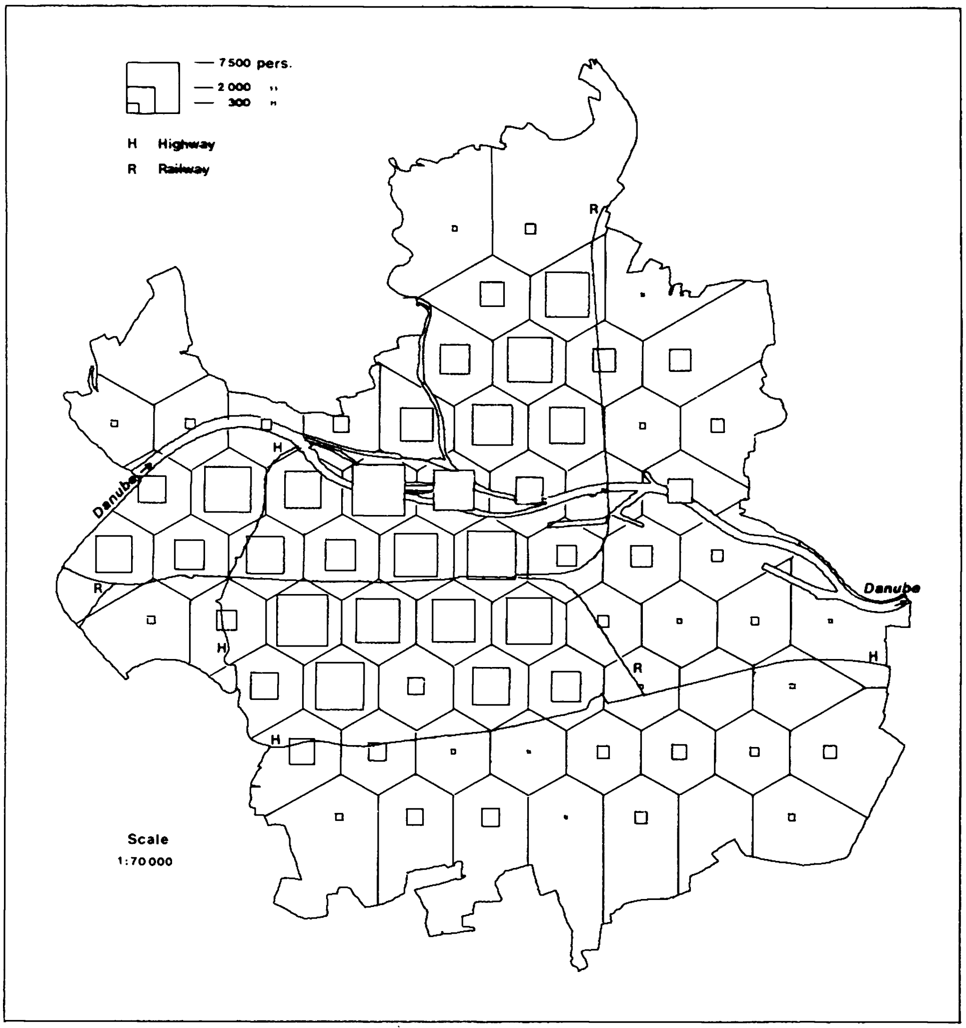

Fig. 3. Regensburg: Aggregation of population for a hexagonal zoning system on the basis of blocks. (Source of data : City Regensburg, Department of Statistics, 1980)

Fig. 2 shows that in the immediate neighbourhood of the chosen centre (x_m, y_m) points will be moved only slightly to avoid a splitting up of the CBD. With increasing distance from city centre and exceeding the densely populated area points will be contracted to even out population in zones. With decreasing population density the original hexagon point pattern undergoes only a slight distortion.
Comparison of Fig. 3 and Fig. 4 indicates that redistribution of population has been successful in city centre and margin, not always at the fringe. The areal variation has been restricted (standard deviation declines from 2265 pers. to 1738 pers.) but no regular pattern is achieved.

What about the impact of point pattern ? Table 2 provides the figures to estimate success of failure of the transformation.

Tab. 2 Measures for the impact of point pattern

	Without barr.		With barr.	
	Hexag.	Transf.	Hexag.	Transf.
	$\cdot 10^4$	$\cdot 10^4$	$\cdot 10^4$	$\cdot 10^4$
$\overline{cov}$	0.673	0.425	0.665	0.460
s_{cov}	1.02	0.581	1.01	0.635
v_{cov}	1.52	1.37	1.52	1.38
$(\alpha=-1.)$	$\cdot 10^{-4}$	$\cdot 10^{-4}$	$\cdot 10^{-4}$	$\cdot 10^{-4}$
$\bar{B}$	1.059	0.832	1.060	0.870
s_B	1.552	1.135	1.552	1.129
v_B	1.47	1.31	1.47	1.30

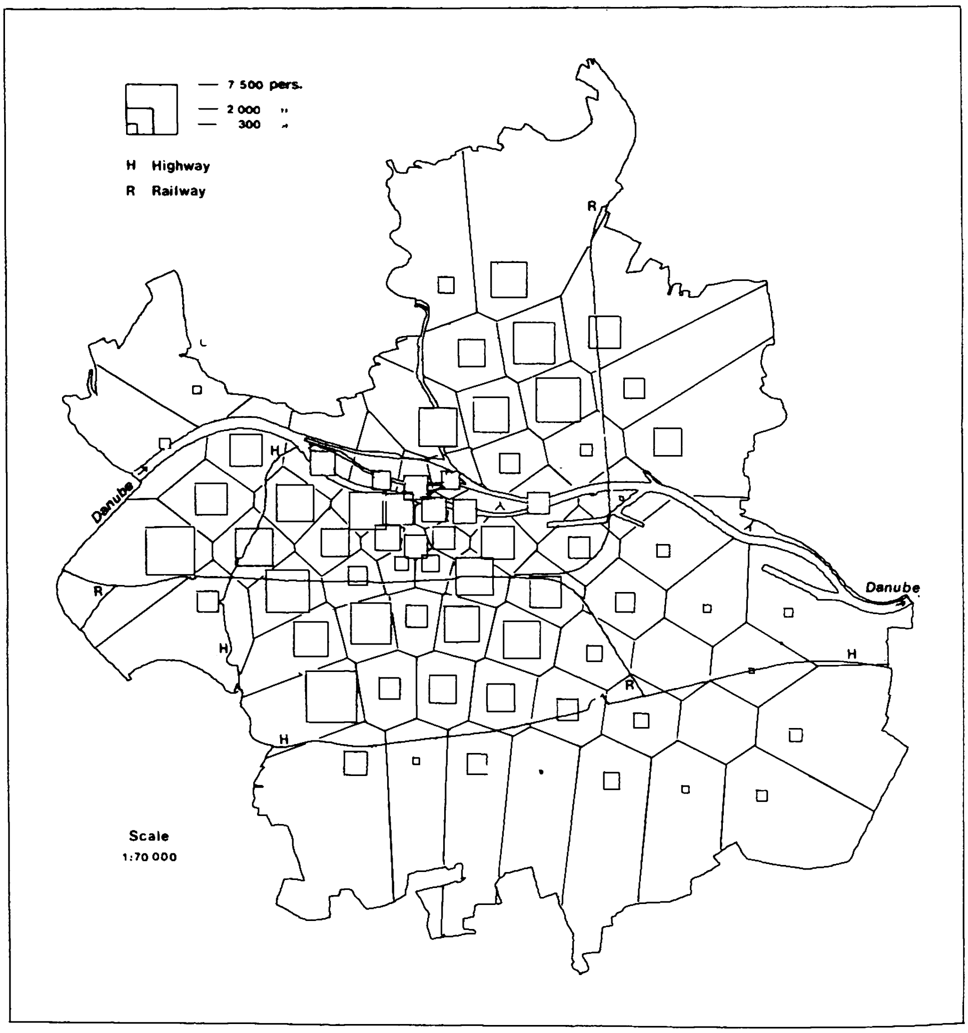

Fig. 4. Regensburg: Aggregation of population for the transformed zoning system on the basis of block data - without consideration of barriers. (Source of data : City Regensburg, Department of Statistics, 1980)

The distortion of the hexagonal zoning system with the consequence of varying distance distribution between sinks will be compensated by the lower variation of V. Therefore cov- as well as B-values fall short of the correspondent values of the hexagonal point pattern. This is valid absolute as well as relative (coefficient of variation $v_{..}$). Note: Completely regular distribution of V could be obtained but at the cost of the still limited variation of point density and in consequence an unbalanced zone shape and area - not only at the fringe like in Fig. 4.

Closer to reality is the assumption that intersection barriers have to be considered. Indeed the rivers Danube and Regen as well as highways and railways affect and hinder the overall interaction. A second computation has been made and the results remain the same (cf. Tab. 1/2). Also the relating of blocks to zone centres within the 11 areas bordered by barriers shows extreme settings only at the fringe (cf. Fig. 5). In most cases the transformed points correspond with the population centre of the zone.

A few zones have been empty before and remain empty after transformation. This is due to the fact that population density is not symmetrical relative to (x_m, y_m). To avoid this and to bring zoning in close accordance with the local conditions it is suggested to depend the transformation function not only on distance but also on direction

$$t = f(r, \varphi) .$$

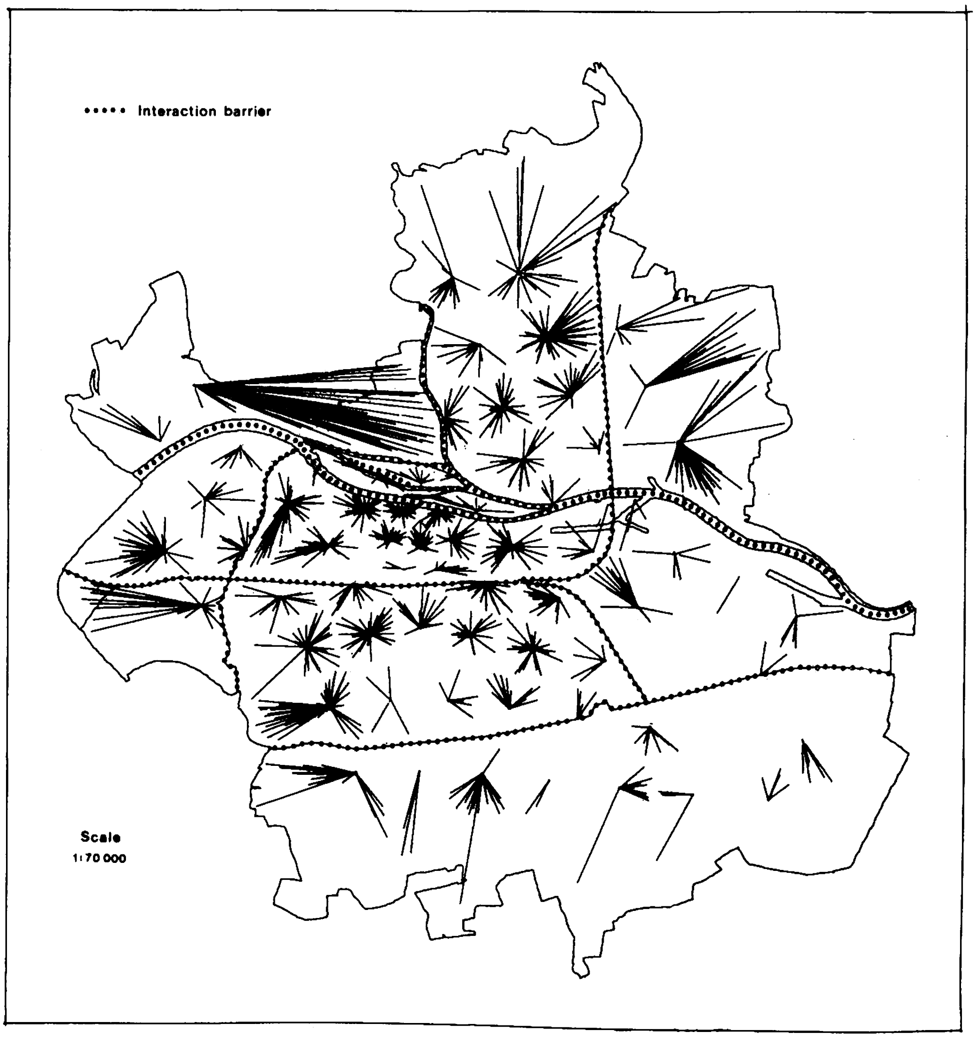

Fig. 5 Regensburg: Aggregation of population for the transformed zoning system on the basis of blocks - with consideration of barriers. Centres of stars are identical with the transformed points, the endpoint of stars are the centres of blocks. (Source of data : City Regensburg, Department of Statistics, 1980)

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