Regensburger
DISKUSSIONSBEITRÄGE
zur Wirtschaftswissenschaft

University of Regensburg Working Papers in Business,
Economics and Management Information Systems

Structural Conditional Correlation

Enzo Weber*

January 02, 2009

Nr. 434

JEL Classification: C32, G10

Keywords: Simultaneity, Identification, EGARCH, Conditional Correlation

* Enzo Weber is Juniorprofessor of Economics at the Department of Economics and Econometrics at the University of Regensburg, 93040 Regensburg, Germany.
Phone: +49-941-943-1952, E-mail: enzo.weber@wiwi.uni-regensburg.de

Structural Conditional Correlation[1]

Enzo Weber

Universität Regensburg
D-93040 Regensburg, Germany
enzo.weber@wiwi.uni-regensburg.de
phone: +49-941-943-1952 fax: +49-941-943-2735

Abstract

A small strand of recent literature is occupied with identifying simultaneity in multiple equation systems through autoregressive conditional heteroscedasticity. Since this approach assumes that the structural innovations are uncorrelated, any contemporaneous connection of the endogenous variables needs to be exclusively explained by mutual spillover effects. In contrast, this paper allows for instantaneous covariances, which become identifiable by imposing the constraint of structural constant / dynamic conditional correlation (SCCC / SDCC). In this, common driving forces can be modelled in addition to simultaneous transmission effects. The methodology is applied to the Dow Jones and Nasdaq Composite indexes, illuminating scope and functioning of the new models.

Keywords: Simultaneity, Identification, EGARCH, Conditional Correlation
JEL classification: C32, G10

[1]This research was supported by the Deutsche Forschungsgemeinschaft through the CRC 649 "Economic Risk". I am grateful to Eric Renault, Jürgen Wolters and Cordelia Thielitz for their comments. Of course, all remaining errors are my own.

1 Introduction

Identifying structural models that feature simultaneous effects between several variables is one of the key tasks of econometrics. In multivariate time series analysis, the conventional method solving identification problems works through parametric (zero) constraints, which allow recovering the structural model from its estimated reduced form. In several research areas, such as monetary economics, considerable progress has been made in theoretically deriving distinct identification schemes. Nonetheless, many economic setups are not compatible with using *a priori* parameter restrictions, since these inherently decide about directions of causality for reasons other than empirical exploration. An example is given in the present paper, which asks provocatively, whether large cap (Dow Jones Industrial Average) or high-tech (Nasdaq Composite) equities predominate the stock segment interdependence, as defined by mutual instantaneous transmission effects. Such a question can evidently not be tackled for instance by imposing a recursive structure on the model.

For heteroscedastic series, some authors in a small recent literature introduced methods that exploit non-constant variances for identifying simultaneous models "through heteroscedasticity" (see Rigobon 2003). A shift in the structural volatility, which yields more additional determining equations from the reduced-form covariance-matrix than unknown coefficients, underlies the basic idea. Building on this logic, pertinent research for example in Sentana and Fiorentini (2001), Rigobon (2002) and Weber (2007a) proposed estimating ARCH-type processes as to coherently describe the necessary volatility movements.

Even though these existing approaches support the identification of unconstrained contemporaneous interactions, they still assume that the structural innovations are uncorrelated. Necessarily, such an assumption explains any correlation of the included variables exclusively by causal transmission effects between them. It follows that in presence of neglected exogenous shocks affecting two variables with equal sign, the estimation is bound to overstate the bilateral causality. In the Dow-Nasdaq example, the fact that both these segments are subject to common news and influences is economically trivial, though econometrically rather intricate. While the problem might in principle be treated by augmenting the model by further essential variables, much relevant information will be unobservable or can hardly be covered in its entirety by necessarily low-dimensional time series systems. In consequence, this discussion stresses the importance of allowing for contemporaneous interaction in the structural innovations.

As a matter of fact, the assumption of uncorrelated structural residuals serves to assure that the additional information obtained from time-varying volatility is not simply exhausted by extra covariance parameters for each regime. That is, introducing unrestricted time-varying covariances would simply undo the identifiability created by heteroscedasticity. Now, the contribution of the present paper lies in rendering instantaneous residual linkage identifiable by extending the idea of constant conditional correlation (CCC) from Bollerslev (1990) to *structural* disturbances, which is mirrored in the name SCCC for the new model. Put differently, time-varying covariances become assessable by restricting them to be governed by the conditional variance dynamics. In order to provide additional flexibility, the SCCC concept is extended to *dynamic* correlations, logically named SDCC.

This methodological approach is discussed at length in the following section. Concerning the empirical US stock example, section 3 reveals a moderate preponderance of the Dow Jones compared to the Nasdaq Composite. The sensitivity of the estimation outcome to sample and model changes is discussed. The last section summarises the key results of the present analysis and proposes further econometric refinements.

2 Methodology

The description of the modelling strategy proceeds in several steps: At first, the structural form of the mean equations is presented, followed by a discussion of inherent identification problems. Thereafter, as constituent parts of an appropriate solution, a multivariate EGARCH process for the conditional variances of the structural residuals, the SCCC specification for the according fundamental correlation as well as the extension to SDCC are exposed. Finally, estimation by Maximum Likelihood is tackled.

2.1 Basic Model and Identification

A simplified model for contemporaneous transmission effects between n endogenous variables y_{it} is specified as

$$Ay_t = \varepsilon_t \; . \tag{1}$$

Here, the coefficients representing instantaneous impacts are included in the matrix A, in which the diagonal elements are normalised to one. ε_t is an n-dimensional vector of structural innovations with unrestricted correlation matrix. Of course, this model can easily be adapted to cover vector autoregressive lags or deterministic terms, as considered in the empirical example in section 3.

For the moment, assume that the ε_{it} were uncorrelated. It is well known that even then, (1) as it stands is not identified and therefore cannot be consistently estimated. The derivation of the reduced-form model simply results in

$$y_t = A^{-1}\varepsilon_t \ . \tag{2}$$

Herein, it naturally proves impossible to recover the structural parameters from the reduced form without further constraints: In the matrix A with normalised diagonal, $n(n-1)$ simultaneous impacts have to be estimated. In (2), this contemporaneous interaction is reflected by cross-correlation of the reduced-form residuals. However, the information contained in the according covariance-matrix is not sufficient for identification, because due to its symmetry, it delivers only $n(n-1)/2$ equations for simultaneous covariances. The n variances are generally needed to balance the same number of their structural counterparts. In the above-mentioned bivariate Dow-Nasdaq example, this leads for instance to a lack of $2(2-1) - 2(2-1)/2 = 1$ equation. Since simply imposing zero constraints in order to reduce the number of parameters shall be excluded as a reasonable strategy, any acceptable solution logically has to augment the number of available determining equations.

The idea of considering such hitherto neglected information motivates the recent literature of so-called "identification through heteroscedasticity" (e.g. Rigobon 2003): For example, assume that it is possible to identify two separate time regimes with differing variances of the structural residuals ε_t, which shall still be uncorrelated. The variance shift between the regimes would deliver two distinct reduced-form covariance-matrices, so that $n(n-1)/2$ additional covariance equations and n additional variance equations could be obtained from the second matrix. Since the number of free parameters only rises by n, the number of structural variances, full identification can be achieved. While time-varying volatility has become a common feature throughout the empirical financial literature, determining a valid date for imposing a single shift in variance is naturally problematic. Therefore, in this point I will follow the econometric procedure in Weber (2007a), who specifies multivariate EGARCH processes for the structural residuals. This basically keeps up the intuition of identification through volatility regimes. Any ARCH-type model however practically defines a distinct variance state for every single observation. This can be thought of as modelling a quasi continuum of regimes, which is reflected in the estimated conditional variances.

Now, recall that the preceding paragraphs have assumed uncorrelated innovations "for the moment". As explained in the introduction, the absence of any common grounds in factor dependence is economically unrealistic; unfortunately however, unrestricted time-varying covariances would lead to the unfavourable situation that each shift in variance introduces as many structural parameters (variances and covariances) as additional equations from the reduced-form covariance-matrices, so that nothing would be gained from heteroscedasticity. In this respect, the current paper makes the contribution of allowing for contemporaneous interaction in the structural disturbances without compromising identifiability. This obvious dilemma is solved by restricting the covariance dynamics of the structural disturbances to result in constant conditional correlations, leading to the name structural CCC (SCCC) with reference to Bollerslev (1990). The idea is that once the constant correlation coefficient is taken into account, shifts in volatility introduce no additional unknown covariance parameters.[2] Furthermore, in order to relax the SCCC assumption in favour of time-varying fundamental correlations, a dynamic model version (SDCC) will be explored.

2.2 Structural EGARCH

In the following, the model setup shall be formalised. First, denote the conditional variances of the elements in ε_t by

$$\mathrm{Var}(\varepsilon_{jt}|I_{t-1}) = h_{jt}^2 \qquad j = 1, \ldots, n \ , \tag{3}$$

where I_{t-1} stands for the whole set of available information at time $t-1$.

Then, stack the conditional variances in the vector $H_t = \begin{pmatrix} h_{1t}^2 & \ldots & h_{nt}^2 \end{pmatrix}'$.

At last, denote the standardised white noise residuals by

$$\tilde{\varepsilon}_{jt} = \varepsilon_{jt}/h_{jt} \qquad j = 1, \ldots, n \ . \tag{4}$$

Then, the multivariate EGARCH(1,1)-process, as suggested by Weber (2007a), is given by

$$\log H_t = C + G \log H_{t-1} + D|\tilde{\varepsilon}_{t-1}| + F\tilde{\varepsilon}_{t-1} \ , \tag{5}$$

where C is a n-dimensional vector of constants and G, D and F are $n \times n$ coefficient matrices. The absolute value operation is to be applied element by element and provides the pure magnitude of shocks. Note that in the original univariate formulation of Nelson (1991), the unsigned shock was corrected for its mean as in $(|\tilde{\varepsilon}_{t-1}| - E(|\tilde{\varepsilon}_{t-1}|))$. The

[2]Qualitatively comparable considerations underlie the structural factor model in Weber (2007b), which parsimoniously models the contemporaneous covariance structure and is applicable to systems of at least three endogenous variables.

present specification merges the term $-D \cdot E(|\tilde{\varepsilon}_{t-1}|)$ into the constants C, but is completely conformable to the original version. The advantage is that no distributional assumption has to be made for calculating the expectation. The signed $\tilde{\varepsilon}_t$ introduce asymmetric volatility effects. In (5), only the conditional variances are modelled, while the covariances are treated in the following section in the context of SCCC.

2.3 Structural Constant Conditional Correlation

The covariances can be recovered by the constant conditional correlation assumption as

$$\mathrm{Cov}(\varepsilon_{it}, \varepsilon_{jt}|I_{t-1}) = h_{ijt} = \rho_{ij} h_{it} h_{jt} \qquad i \neq j \ , \tag{6}$$

where ρ_{ij} denotes the correlation between the ith and jth residual.

Let R designate the correlation matrix of ε_t, holding ones on the main diagonal and the ρ_{ij} as its off-diagonal elements. Then, the conditional covariance matrix Ω_t of the structural innovations results as

$$\Omega_t = diag\{H_t\}^{1/2} R \, diag\{H_t\}^{1/2} \ . \tag{7}$$

Accounting for the discussion in Bollerslev (1990) and given positive variances from the log-linearised EGARCH, Ω_t is assured to be positive definite.

This property carries over to the conditional covariance-matrix of the reduced-form residuals $A^{-1}\varepsilon_t$

$$\Sigma_t = A^{-1} \Omega_t (A^{-1})' \tag{8}$$

due to its quadratic form. Cross-correlations, as represented by non-zero off-diagonal elements, can arise both from spillovers according to the coefficients in A^{-1} or from structural covariances h_{ijt} (the off-diagonal entries in Ω_t). In this context, note as well that the constant correlation restriction only applies to the *structural* innovations; the realised variables y_{it} may well feature time-varying correlation depending on the variance developments and the spillovers in A. Nonetheless, SCCC remains an assumption that has to be tackled empirically. For this reason, the following section will extend the model towards allowing for time-varying fundamental correlations.

Building on the preceding elaboration, identifiability can now be discussed concretely for the given model. For the sake of illustration and without loss of generality, I focus on the bivariate case, which is directly applicable to the Dow-Nasdaq example. The structural variance process (5) contains two parameters in C and four each in G, D and F. Together with the two parameters from the structural matrix A and one from ρ, the sum adds up to 17 coefficients. This can be compared to the number arising from a reduced-form process for $vech(\Sigma_t)$, where the *vech* operator stacks the lower triangular portion of a matrix into a column vector. For the given example, this vector includes two variances and one covariance. Thus, in a general MGARCH, the equivalent of C has dimension 3×1 and those of G, D and F are 3×3. Consequently, the number of parameters arrives at a total of $3 + 3\,(3 \cdot 3) = 30$, which exceeds 17 and hence would satisfy the necessary summing-up constraint. Note that an unrestricted MGARCH for the *structural* residuals would be of the same shape as the one for $vech(\Sigma_t)$, so that no additional information would be obtained to recover the coefficients in A. In the parlance of volatility regimes, this corresponds to using up all additional determining equations for unconstrained covariance parameters.

In addition, a sufficient condition is given by linear independence of the conditional variances, similar to Sentana and Fiorentini (2001). To clarify this point in the current context, imagine that in the bivariate example, the first variance would be proportional with parameter b to the second variance: $h_{1t}^2 = b h_{2t}^2$. From the SCCC assumption (6), it would follow that $h_{12t} = \rho\sqrt{b} h_{2t}^2$. Hence, all elements in the structural covariance matrix Ω_t would equal some multiple of h_{2t}^2. The same would hold for the reduced-form covariance matrix Σ_t, since by (8), its entries are simply linear combinations of the Ω_t elements. Logically, the dimension of the process for $vech(\Sigma_t)$ would collapse, leaving no additional information in comparison to the structural variance equation. In conclusion, there would be no unique way to determine the contributions of the structural coefficients ρ, A_{12} and A_{21} to the second moments. I will shortly return to this issue in the empirical application.

2.4 Structural Dynamic Conditional Correlation

As mentioned above, return correlation can emerge from spillovers or structural correlation. Under the SCCC assumption of constant fundamental correlations, the time-varying part of the total correlation has to be picked up exclusively by the mutual transmission effects. Consequently, above all in times of economic turbulences, the estimation might easily understate the influence of third-party common factors, as far as it is not constant. This suggests that the following dynamic model extension (SDCC) should be able to compensate for the depicted shortcoming.

Building on Engle (2002), define the new conditional correlation matrix R_t as

$$R_t = diag\{Q_t\}^{-1/2} Q_t \, diag\{Q_t\}^{-1/2} \; . \tag{9}$$

Therein, let Q_t follow the process

$$Q_t = (1 - \alpha - \beta)\overline{Q} + \alpha\tilde{\varepsilon}_{t-1}\tilde{\varepsilon}'_{t-1} + \beta Q_{t-1} \ . \tag{10}$$

(10) corresponds to a standard GARCH(1,1) in that Q_t is driven by the cross product of the shocks and a persistence term. $\overline{Q}$ denotes the unconditional covariance matrix of the standardised residuals $\tilde{\varepsilon}_t$. Although α and β are defined as scalars for parsimony, more comprehensive solutions are possible, see Engle (2002).

With R_t at hand, the conditional covariance-matrix Ω_t of the structural disturbances ε_t is defined as

$$\Omega_t = diag\{H_t\}^{1/2} R_t \, diag\{H_t\}^{1/2} \ . \tag{11}$$

For reasons explained in Engle (2002), Ω_t is assured to be positive definite. Again, the same holds for the reduced-form conditional covariance-matrix Σ_t, see (8).

Discussion of identifiability follows the lines from section 2.3. The SCCC coefficient ρ is now replaced by the off-diagonal element from $\overline{Q}$, in addition to α and β. In sum, 19 coefficients have to be estimated, what is evidently still feasible. To summarise the main facts, the SCCC held up identifiability in presence of time-varying covariances by requiring them to follow the product of the standard errors. Going one step further, SDCC can even allow for time-varying correlations by requiring them to follow the parsimonious, but still flexible, process (10).

2.5 Estimation

The estimation can be done by Maximum Likelihood. For this purpose, the log-likelihood for a sample of T observations (complemented by an adequate number of pre-sample observations) under the assumption of conditional normality is constructed as

$$L(\theta) = -\frac{1}{2}\sum_{t=1}^{T}(n \log 2\pi + \log|\Sigma_t| + y'_t\Sigma_t^{-1}y_t) \ , \tag{12}$$

where the vector θ stacks all free parameters from C, G, D, F, A and ρ, respectively α, β and $\overline{Q}$ for SDCC. That is, maximisation of (12) yields estimates of both the EGARCH parameters and the structural coefficients governing spillovers and fundamental correlations.

As the assumption of conditional normality is usually problematic for financial markets data, the estimation relies on Quasi-Maximum-Likelihood (QML, see Bollerslev and

Wooldridge 1992). This ensures consistency of the estimation, while standard errors are corrected for possible non-normality. Furthermore, results will be compared to estimates obtained under more fat-tailed distributions.

Common-practice simplifications like correlation and variance targeting or two-stage estimation (e.g., Engle 2002) do not seem possible in the current structural context. For example, the correlation matrices R or $\overline{Q}$ are not identified *a priori* from the unconditional distribution. The same is true for the structural shocks, so that the EGARCH processes cannot be separately estimated in advance. Consequently, one-step QML is pursued, adopting the BHHH algorithm (Berndt et al. 1974) for numerical likelihood optimisation.

3 Blue Chip vs. High Tech

3.1 Data

Several times, the preceding discussion has already recurred to the exemplary experiment of analysing mutual and common influences between the Dow Jones Industrial Average and the Nasdaq Composite. Here, the sample of daily returns begins on 2/5/1971, where Nasdaq had started, and ends on 10/31/2007; data source is Reuters. Figure 1 presents the return series and the well-known picture of the index development. Most eye-catching are the Black Monday in 1987 and the extremely volatile period around 2000, where stock prices fell due to the "new economy" bubble burst and the general recession.

In preparation for the empirical procedure, the returns were filtered by regressing them on a constant and four day-of-the-week dummies. Based on the suggestion of the Bayesian information criterion, autoregressive lags were not included.

3.2 SCCC Results

For the optimisation of the likelihood (12), starting values were obtained as follows: The EGARCH parameters were estimated in univariate models, whereas the off-diagonal elements were set to zero. The variance processes were started at the sample moments. A was initialised as the identity matrix, so that ρ equalled the unconditional return correlation; putting more weight on A and less on ρ had no relevant impact on the outcome of the QML procedure. The estimations were carried out in a Gauss programme employing

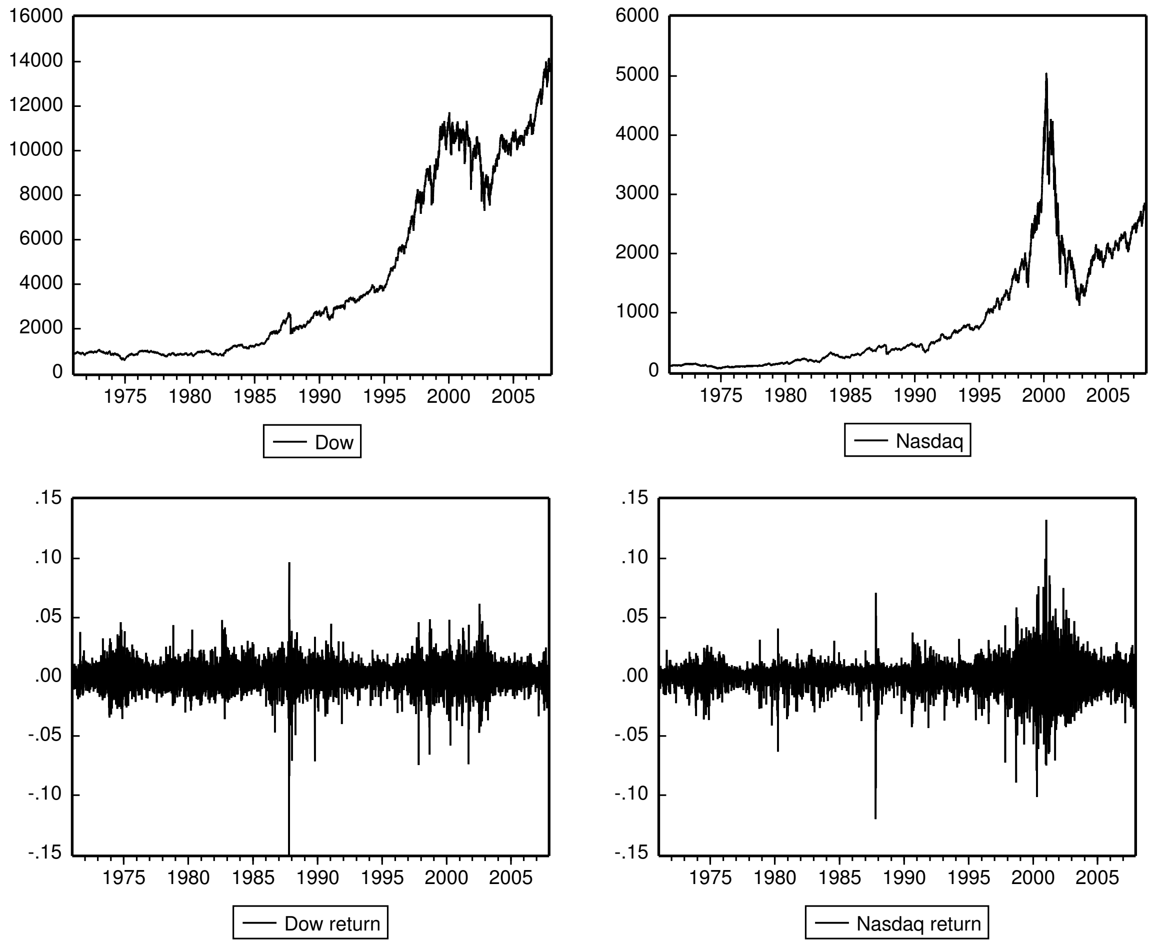

Figure 1: Dow Jones and Nasdaq Composite indexes and returns

the CML module.

Equations (13) and (14) display the results for the mean and variance models. The variable names denote close-to-close returns at time t and QML standard errors are in parentheses.

$$\begin{aligned} \mathrm{DJIA}_t &= \underset{(0.035)}{0.309}\,\mathrm{NQC}_t + \hat{\varepsilon}_{1t} \\ \mathrm{NQC}_t &= \underset{(0.024)}{0.402}\,\mathrm{DJIA}_t + \hat{\varepsilon}_{2t} \\ \hat{\rho} &= \underset{(0.068)}{0.196} \end{aligned} \tag{13}$$

$$\begin{pmatrix} \log h_{1t}^2 \\ \log h_{2t}^2 \end{pmatrix} = \begin{pmatrix} \underset{(0.029)}{-0.167} \\ \underset{(0.024)}{-0.252} \end{pmatrix} + \begin{pmatrix} \underset{(0.005)}{0.984} & \underset{(0.002)}{-0.007} \\ \underset{(0.006)}{-0.021} & \underset{(0.003)}{0.989} \end{pmatrix} \begin{pmatrix} \log h_{1t-1}^2 \\ \log h_{2t-1}^2 \end{pmatrix} + \begin{pmatrix} \underset{(0.023)}{0.130} & \underset{(0.017)}{0.067} \\ \underset{(0.019)}{0.125} & \underset{(0.018)}{0.171} \end{pmatrix} \begin{pmatrix} |\tilde{\varepsilon}_{1t-1}| \\ |\tilde{\varepsilon}_{2t-1}| \end{pmatrix} + \begin{pmatrix} \underset{(0.011)}{-0.041} & \underset{(0.006)}{-0.029} \\ \underset{(0.010)}{-0.051} & \underset{(0.007)}{-0.035} \end{pmatrix} \begin{pmatrix} \tilde{\varepsilon}_{1t-1} \\ \tilde{\varepsilon}_{2t-1} \end{pmatrix} \tag{14}$$

First of all, the mean equations deliver highly significant spillover effects in both directions. These confirm *a priori* expectations in a sense that the blue chip index Dow Jones dominates the mutual transmission, even if only moderately. Reasons for such spillovers might be found in stock price signalling, wealth and liquidity effects, cross-market hedging, herding behaviour or market microstructure. Furthermore, the benefit gained from the new SCCC model is verified by the estimate for ρ, which is highly significant when related to its standard error as well as in a system likelihood ratio test with $H_0 : \rho = 0$.

The variance model (14) is in line with (13) in that the more sizeable spillovers originate in the Dow Jones. In the volatility domain, such transmission effects can economically be ascribed the role of a proxy for information flows between markets (Ross 1989). The negative parameters of the signed shocks represent the well-known asymmetric volatility effects. The negative off-diagonal coefficients in the autoregressive matrix indicate a certain dampening influence, which is however economically small.[3] Being smaller than one, both eigenvalues of this matrix meet the stability criterion, even though the usual substantial persistence in variance can be found. The empirical correlation of the structural variances amounts to 29%, so that there is evidently no problem with linear dependence, which would sacrifice identification. Figure 2 depicts the log-variances.

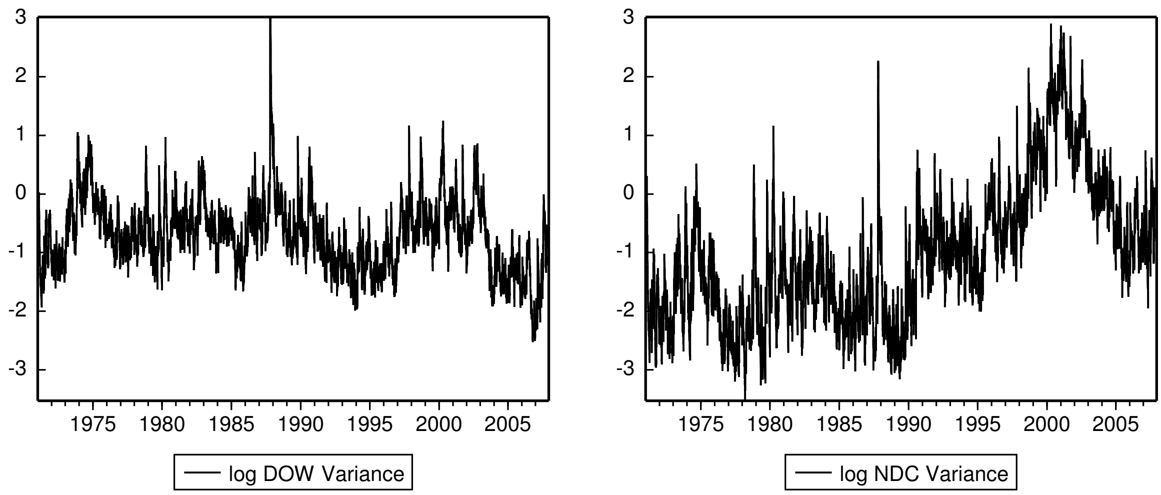

Figure 2: Dow Jones and Nasdaq Composite structural log-variances

As a test for appropriate model specification, the autocorrelations of the squared standardised disturbances $\tilde{\varepsilon}_{jt}^2$ were checked. The approximate 95% confidence bands are never exceeded but by the Nasdaq first-order autocorrelation, which does however not reach significance at the 1% level. In general, this confirms the common results in the literature,

[3]For example, consider a structural unit shock in the Dow, which raises both Dow and Nasdaq variance. In the subsequent period, the increased Dow variance negatively affects the Nasdaq variance, so that the initial spillover effect is dampened. Calculating the dynamics though several periods would result in variance profiles.

which state that GARCH-type models of orders $1,1$ are fairly appropriate for financial markets data. The same conclusions are reached when standardising the reduced-form returns y_{jt} by the variances from (8) instead of the structural innovations and when applying formal LM tests for remaining ARCH. Furthermore, it is checked whether the conditional covariance under the constant correlation assumption adequately reflects the dynamics of the comovement of the structural innovations. For this purpose, I consider the autocorrelations of the cross products of the standardised residuals $\tilde{\varepsilon}_{1t}\tilde{\varepsilon}_{2t}$:[4] Even though these serial correlations were greatly reduced when compared to the raw data, clearly significant values are left. This fact naturally directs the focus of the next step towards the appropriateness of the SCCC assumption.

A fundamental correlation of $\hat{\rho} = 20\%$ might appear to low for two leading US-based stock segments, especially given the overall unconditional correlation of 69%. As shown by Figure 1, at least the picture of the Nasdaq returns is largely dominated by the volatility generated by bubble and subsequent breakdown in the technology market. In addition, the unsatisfactory outcome of the full-sample specification test suggests that SCCC fails to capture the covariation dynamics over the whole range. Therefore, the estimation procedure is re-run with a sample cut at the end of 1996.[5] The results are as follows:

$$\begin{aligned} \mathrm{DJIA}_t &= \underset{(0.069)}{0.154}\ \mathrm{NQC}_t + \hat{\varepsilon}_{1t} \\ \mathrm{NQC}_t &= \underset{(0.033)}{0.325}\ \mathrm{DJIA}_t + \hat{\varepsilon}_{2t} \\ \hat{\rho} &= \underset{(0.078)}{0.423} \end{aligned} \tag{15}$$

$$\begin{pmatrix} \log h^2_{1t} \\ \log h^2_{2t} \end{pmatrix} = \begin{pmatrix} \underset{(0.049)}{-0.208} \\ \underset{(0.037)}{-0.338} \end{pmatrix} + \begin{pmatrix} \underset{(0.009)}{0.975} & \underset{(0.008)}{-0.025} \\ \underset{(0.011)}{-0.040} & \underset{(0.009)}{0.959} \end{pmatrix} \begin{pmatrix} \log h^2_{1t-1} \\ \log h^2_{2t-1} \end{pmatrix} + \begin{pmatrix} \underset{(0.028)}{0.119} & \underset{(0.030)}{0.092} \\ \underset{(0.026)}{0.128} & \underset{(0.026)}{0.214} \end{pmatrix} \begin{pmatrix} |\tilde{\varepsilon}_{1t-1}| \\ |\tilde{\varepsilon}_{2t-1}| \end{pmatrix} + \begin{pmatrix} \underset{(0.013)}{-0.028} & \underset{(0.009)}{-0.029} \\ \underset{(0.015)}{-0.055} & \underset{(0.011)}{-0.041} \end{pmatrix} \begin{pmatrix} \tilde{\varepsilon}_{1t-1} \\ \tilde{\varepsilon}_{2t-1} \end{pmatrix} \tag{16}$$

In comparison to model (13), the correlation coefficient has more than doubled to 42%, which is clearly more in line with *a priori* expectations. A rise in the sample correlation that might explain this effect does not occur. Rather, the considerable change due to the sample shortening suggests that the extremely volatile period around 2000 is obviously not compatible with time-invariant correlation of the fundamental innovations. In particular, the widely celebrated shift from the "old" to the "new" economy sectors as well as the

[4]See Bollerslev (1990). Note that by (6), the conditional covariance is proportional to the product of the standard errors. While the chosen test is straightforward and widely used standard, it might nonetheless be overly simplistic. Therefore, I additionally applied the more elaborate procedure from Engle and Sheppard (2001), which confirms the impression from the basic autocorrelation tests.

[5]Cutting the sample in the earlier 1990s does not produce qualitative differences.

sudden reversal interfered the afore prevailing stable pattern. This economic intuition can be econometrically supported by checking whether in contrast to the full-sample estimation, the standardised cross products are now free of autocorrelation: Indeed, $\tilde{\varepsilon}_{1t}\tilde{\varepsilon}_{2t}$ turns out as pure white noise in the version cut in 1996.

Interestingly, the cross products $y_{1t}y_{2t}$ of the *reduced-form* returns, standardised by the according conditional covariances from (8), reveal no remaining serial correlation even in the full sample. This result is perfectly in line with the discussion of the merits of non-constant structural correlation from section 2.4: Obviously, the time variation in the overall conditional correlations is picked up by the spillovers, producing a proper empirical fit of the *reduced-form* conditional covariance. However, the assumption of the underlying *structural* CCC can still fail, leading to underestimation of ρ. After all, the key feature of the first estimation carries over to the outcome in the smaller sample: The Dow still dominates the mutual transmission effects, uniformly in the mean and variance domain.

The standardised residuals have excess kurtosis 4.4 and 1.5, respectively. On the one hand, this substantiates the decision of using QML standard errors, but on the other hand, more heavy tailed distributions may be applied. The GARCH literature (e.g. Carnero et al. 2004) has established a tight connection between persistence and kurtosis. Logically, changing the latter affects the heteroscedasticity, which serves identification in the present context. To affirm robustness in this direction, I re-estimated the model under a Student-t-likelihood. Deviations from (13) and (14) were indeed numerically negligible.

As a further noteworthy point, let us address the role of the volatility spillovers, which represent a special feature of my particular mean-variance multiple equation setup. Restricting the off-diagonal elements in G, D and F to zero leaves two non-interacting volatility processes. Such diagonal models have become quite popular in the multivariate GARCH literature. In the current context, this corresponds to identifying the simultaneity by univariate structural GARCH. Imposing the constraints (that are of course clearly rejected), the according *full-sample* estimation results in

$$\begin{aligned} \mathrm{DJIA}_t &= \underset{(0.030)}{0.183}\ \mathrm{NQC}_t + \hat{\varepsilon}_{1t} \\ \mathrm{NQC}_t &= \underset{(0.029)}{0.290}\ \mathrm{DJIA}_t + \hat{\varepsilon}_{2t} \\ \hat{\rho} &= \underset{(0.052)}{0.456} \end{aligned} \tag{17}$$

$$\begin{pmatrix} \log h^2_{1t} \\ \log h^2_{2t} \end{pmatrix} = \begin{pmatrix} \underset{(0.020)}{-0.110} \\ \underset{(0.017)}{-0.172} \end{pmatrix} + \begin{pmatrix} \underset{(0.004)}{0.986} & 0 \\ 0 & \underset{(0.002)}{0.986} \end{pmatrix} \begin{pmatrix} \log h^2_{1t-1} \\ \log h^2_{2t-1} \end{pmatrix} + \begin{pmatrix} \underset{(0.026)}{0.134} & 0 \\ 0 & \underset{(0.020)}{0.213} \end{pmatrix} \begin{pmatrix} |\tilde{\varepsilon}_{1t-1}| \\ |\tilde{\varepsilon}_{2t-1}| \end{pmatrix} + \begin{pmatrix} \underset{(0.014)}{-0.043} & 0 \\ 0 & \underset{(0.007)}{-0.040} \end{pmatrix} \begin{pmatrix} \tilde{\varepsilon}_{1t-1} \\ \tilde{\varepsilon}_{2t-1} \end{pmatrix} . \tag{18}$$

Equation (17) shows a $\hat{\rho}$ that has risen by more than 100%. Qualitatively, the same holds for corresponding results in the shorter sample, which are available upon request. Logically, it can be argued that recurring to simple univariate GARCH processes in identifying simultaneous systems may lead to questionable parameter estimates. In particular, imposing parametric zero restrictions on the variance model in order to shift the identification problem from the mean to the volatility domain cannot be regarded as a virtually innocent strategy. Nevertheless, note that the predominance of the Dow is still preserved in the constrained system. In conclusion, the SCCC model yields solid and well-interpretable results, even if caution is advised in applying the SCCC assumption and in adopting adequate conditional volatility specifications.

3.3 SDCC Results

The difference in SCCC between the full and the shortened sample has been ascribed to the extremely volatile period around the year 2000. As explained in section 2.4, violation of the SCCC assumption can account for underestimation of the fundamental correlation. In view of this problem, the SDCC model is likely to provide a more appropriate impression of the underlying financial processes.

Starting values were determined as before, while for the SDCC coefficients from (10), they were taken from a conventional reduced-form DCC estimation. Equations (19), (20) and (21) display the estimation outcome. Therein, q_t denotes the off-diagonal element from Q_t.

$$\begin{aligned} \text{DJIA}_t &= \underset{(0.036)}{0.326}\,\text{NQC}_t + \hat{\varepsilon}_{1t} \\ \text{NQC}_t &= \underset{(0.037)}{0.328}\,\text{DJIA}_t + \hat{\varepsilon}_{2t} \end{aligned} \tag{19}$$

$$\begin{pmatrix} \log h^2_{1t} \\ \log h^2_{2t} \end{pmatrix} = \begin{pmatrix} \underset{(0.028)}{-0.168} \\ \underset{(0.025)}{-0.247} \end{pmatrix} + \begin{pmatrix} \underset{(0.005)}{0.984} & \underset{(0.003)}{-0.008} \\ \underset{(0.006)}{-0.020} & \underset{(0.003)}{0.987} \end{pmatrix} \begin{pmatrix} \log h^2_{1t-1} \\ \log h^2_{2t-1} \end{pmatrix} + \begin{pmatrix} \underset{(0.021)}{0.126} & \underset{(0.018)}{0.070} \\ \underset{(0.019)}{0.114} & \underset{(0.021)}{0.176} \end{pmatrix} \begin{pmatrix} |\tilde{\varepsilon}_{1t-1}| \\ |\tilde{\varepsilon}_{2t-1}| \end{pmatrix} + \begin{pmatrix} \underset{(0.010)}{-0.036} & \underset{(0.007)}{-0.030} \\ \underset{(0.010)}{-0.045} & \underset{(0.008)}{-0.039} \end{pmatrix} \begin{pmatrix} \tilde{\varepsilon}_{1t-1} \\ \tilde{\varepsilon}_{2t-1} \end{pmatrix} \tag{20}$$

$$q_t = (1 - \underset{(0.006)}{0.021} - \underset{(0.008)}{0.973}) \cdot \underset{(0.081)}{0.361} + \underset{(0.006)}{0.021}\tilde{\varepsilon}_{1t-1}\tilde{\varepsilon}_{2t-1} + \underset{(0.008)}{0.973}q_{t-1} \tag{21}$$

The unconditional correlation of the structural innovations rises to more than 1/3, compared to less than 1/5 in the SCCC model. This confirms the presumption that a higher degree of coherence in shocks can be found by allowing for time variation, which is picked up by the SDCC specification. While in the SCCC model, a relatively high correlation coefficient was only obtained after sample shortening, the dynamic approach achieves such a result over the whole sample including the period of economic and financial disturbances. Logically, the SDCC parameters $\hat{\alpha}$ and $\hat{\beta}$ are clearly significant, taking values that are common in the financial volatility literature. Restricting both of them to zero, that is applying the SCCC assumption, is clearly rejected with a decline in log-likelihood of 225.

Figure 3 shows the structural conditional correlation as the off-diagonal element in R_t as well as its reduced-form counterpart, which is calculated from the covariance-matrix Σ_t. That is, the latter mirrors the correlation effects of causal transmission in addition to the fundamental commonalities in the structural innovations.

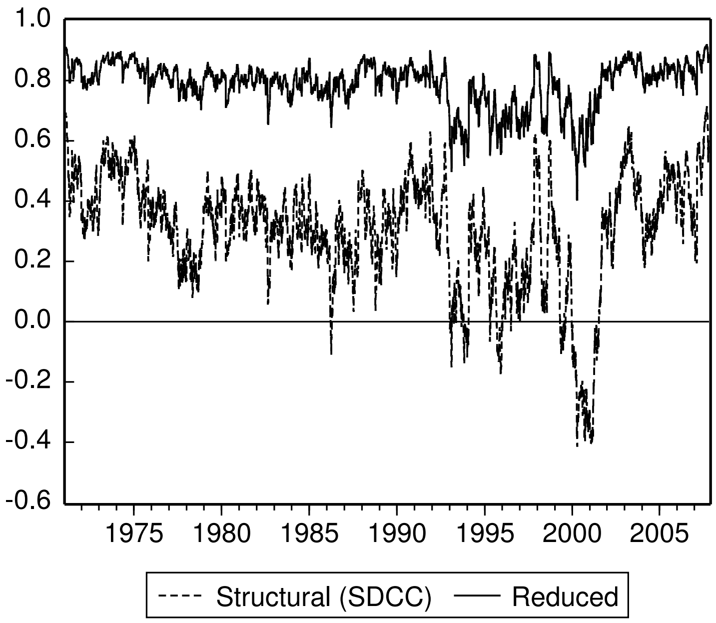

Figure 3: Conditional correlations

The most eye-catching drop in correlations appears in the year 2000, where the extreme spike and fall of the Nasdaq Composite index occurred. Thereafter, correlations jump up again coinciding with 9/11 and the US recession. In the years before, further turbulences took place from the end of 1992 onwards, comprising numerous financial crises like those in Mexico, South-East Asia, Russia and Brazil. A similar pattern has as well been found by Engle (2002) in his reduced-form DCC approach within a shorter sample.

Concerning the direct spillovers, both coefficients in A are highly significant. Since the Dow effect is only marginally higher, the moderate dominance of the Dow found before hardly carries over to the SDCC model. Nevertheless, the causality-in-variance effects from (20) still reveal higher cross-segment influences of the Dow Jones as compared to the Nasdaq Composite.

Results for the autocorrelations of the squared standardised disturbances $\tilde{\varepsilon}_{jt}^2$ are as before. Interestingly, the autocorrelations of the cross product $\varepsilon_{1t}\varepsilon_{2t}$, standardised by the conditional covariance, are insignificant, in contrast to the SCCC model, which could not absorb the whole time variation in the structural covariance. This can be taken as additional evidence that the volatile years around 2000 witnessed a change in the fundamental market correlation, which was properly picked up by the SDCC feature.

4 Concluding Summary

Finding evidence for truly causal relations and distinguishing them from comovement based on third-party influences is a recurrent theme throughout many empirical economic analyses. Omnipresent identification problems often restrain the possibilities of estimating models featuring distinct structural interpretation. Recent progress came through literature contributions that exploit (autoregressive conditional) heteroscedasticity in the data in order to obtain the additional information needed for identifying simultaneous systems.

As a shortcoming, these existing approaches rely on assuming that the structural shocks are instantaneously uncorrelated. Consequently, the observed correlation between the included variables is to its full extent explained by mutual causal transmission effects. Obviously, this neglects any model-exogenous factors, which drive all the endogenous variables alike. Since the presence of such driving forces is economically straightforward, the estimation is bound to overstate the bilateral spillovers. Unfortunately, allowing for unrestricted covariance dynamics violates the conditions for identification.

This paper proposed a method of introducing contemporaneous residual interaction, whilst upholding identifiability. In detail, the dilemma is solved by allowing for covariances that develop in accordance with the new concept of structural constant conditional correlation (SCCC). This approach neither imposes zero or constant covariances nor does it imply constant correlation of the observed reduced-form disturbances. Nevertheless, flexibility was additionally increased by introducing time variation in the fundamental correlations in the SDCC model. The usefulness of the methodology was demonstrated in an example including returns of the Dow Jones and Nasdaq Composite indexes. As might have been expected, the Nasdaq was subject to higher cross-market impacts than the Dow. Furthermore, substantial instantaneous correlation of the structural innovations has been established. In this context, the study was complemented by an insightful discussion of the important role of causality-in-variance effects, the scope of the SCCC assumption and the role of dynamically developing common factor exposure.

In principle, the new methodology could be explicitly qualified by Monte Carlo experiments. I have abstained from elaborating comprehensive results, simply because sufficiently iterated QML procedures would accumulate a prohibitive computation time. However, small-scale simulations were quite encouraging in that the correlation and spillover parameters showed usual deviations of no more than a few percent. Refining the practical estimation algorithm may bring the potential of further inspection to the fore. Moreover, matching close-to-close results to the outcome of high-frequency examinations might represent a promising way to shed light on the sequence of structural processes underlying the herein before proposed identification scheme.

In future research, the econometric progress of the structural conditional correlation models might be further exploited for finding economic interpretations of structural systems, which have hitherto been treated in reduced form. By the same token, existing identification schemes could be checked for their consistency with empirical data. In particular, this study provided a powerful tool for discriminating direct transmission from common factor influence.

References

[1] Berndt, E., B. Hall, R. Hall, J. Hausman (1974): Estimation and Inference in Nonlinear Structural Models. Annals of Social Measurement, 3, 653-665.

[2] Bollerslev, T. (1990): Modeling the coherence in short-run nominal exchange rates: A multivariate generalized ARCH model. Review of Economics and Statistics, 72, 498-505.

[3] Bollerslev, T., J.M. Wooldridge (1992): Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models with Time Varying Covariances. Econometric Reviews, 11, 143-172.

[4] Carnero, M.A., D. Peña, E. Ruiz (2004): Persistence and Kurtosis in GARCH and Stochastic Volatility Models. Journal of Financial Econometrics, 2, 319-342.

[5] Engle, R.F. (2002): Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models. Journal of Business & Economic Statistics, 20, 339-50.

[6] Engle, R.F., K. Sheppard (2001): Theoretical and empirical properties of dynamic conditional correlation multivariate GARCH. NBER Working Paper 8554.

[7] Nelson, D.B. (1991): Conditional Heteroskedasticity in Asset Returns: A New Approach. Econometrica, 59, 347-370.

[8] Rigobon, R. (2002): The Curse of Non-Investment Grade Countries. Journal of Development Economics, 69, 423-449.

[9] Rigobon, R. (2003): Identification through heteroscedasticity. Review of Economics and Statistics, 85, 777-792.

[10] Ross, S.A. (1989): Information and volatility: the no-arbitrage martingale approach to timing and resolution irrelevancy. Journal of Finance, 44, 1-17.

[11] Sentana, E., G. Fiorentini (2001): Identification, estimation and testing of conditionally heteroskedastic factor models. Journal of Econometrics, 102, 143–164.

[12] Weber, E. (2007a): Volatility and Causality in Asia Pacific Financial Markets. CRC 649 Discussion Paper 2007-004, Humboldt-Universität zu Berlin.

[13] Weber, E. (2007b): Correlation vs. Causality in Stock Market Comovement. CRC 649 Discussion Paper 2007-064, Humboldt-Universität zu Berlin.