The pricing of temperature futures at the Chicago Mercantile Exchange

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Agenda

1  Index Modeling

2  Modeling market prices

3  Trading strategies

4  Conclusion
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1 Index Modeling

2 Modeling market prices

3 Trading strategies

4 Conclusion
Index Modeling

- Uses historical index values
  - Prediction of the expected index $Y_{n+1}$ as the mean of the preceding $n$ years, or
  - Using the preceding $n$ years and predict the expected index $Y_{n+1}$ with an extrapolation of a linear trend
- Calculation of the uncertainty of the prediction using the theory of linear models

**Expectations Hypothesis**

The futures price $F_t$ is given by

$$0 = E^Q_t(Y_{n+1} - F_t) = E^O_t(Y_{n+1}) - F_t = E^P_t(Y_{n+1}) - F_t$$
Index Modeling – Error estimation

Assumptions

- Existence of a linear trend
- Errors are i.i.n.d.

Linear detrending

$$\text{MSE}(\hat{y}_0 + \epsilon_0) = \frac{(n+2)(n+1)}{n(n-1)} \sigma^2$$
$$\text{ME}(\hat{y}_0 + \epsilon_0) = 0$$

$$\sigma^2 = \text{real variance}$$

No detrending

$$\text{MSE}(\hat{y}_0 + \epsilon_0) = \beta^2 \left(\frac{n+1}{2}\right)^2 + \frac{n+1}{n} \sigma^2$$
$$\text{ME}(\hat{y}_0 + \epsilon_0) = \beta^2 \left(\frac{n+1}{2}\right)^2$$

$$\sigma^2 = \text{real variance}$$
$$\beta = \text{real trend}$$
Index Modeling – Error estimation

Root Mean Square Error of Index Modeling (virtual contracts)

- RMSE Linear Detrend
- RMSE Flat Line

Number of years

RMSE/σ

β = 0.05σ
β = 0.04σ
β = 0.03σ
β = 0.02σ
β = 0.01σ
β = 0
Index Modeling – Error estimation

Root Mean Square Error of Index Modeling (virtual contracts)

Number of years

\[
\text{RMSE} / \sigma
\]

\[
\beta = 0.05\sigma
\]

\[
\beta = 0.04\sigma
\]

\[
\beta = 0.03\sigma
\]

\[
\beta = 0.02\sigma
\]

\[
\beta = 0.01\sigma
\]

\[
\beta = 0
\]

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Index Modeling – Error estimation

Mean Error of Index Modeling (virtual contracts)

-50 0 50 100
β = 3.5
β = 3
β = 2.5
β = 2
β = 1.5
β = 1
β = 0.5
β = 0
β = -0.5
β = -1
β = -1.5
β = -2

ME Linear Detrend
ME Flat Line

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Index Modeling – Error estimation

Mean Error of Index Modeling (virtual contracts)

Number of years

Mean error

β = 3.5
β = 3
β = 2.5
β = 2
β = 1.5
β = 1
β = 0.5
β = 0
β = -0.5
β = -1
β = -1.5
β = -2

HDD

CDD

ME Linear Detrend
ME Flat Line
Real ME Linear Detrend
Real ME Flat Line
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Dataset

- Daily settlement prices of CME temperature futures
- September 2003 – April 2006
  - 3 winter seasons, 2 summer seasons
- Seasonal and monthly contracts
- 18 US weather stations

Total of approx. 15,000 prices
Chicago seasonal HDD 2005/06

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Chicago seasonal HDD 2005/06

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Chicago seasonal HDD 2005/06

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Index Modeling including weather forecasts

- Pricing formula:

\[ I^n_t(\tau_1, \tau_2) = \sum_{k=\tau_1}^{t-1} T_{k,n+1} + \sum_{k=t}^{t+7} \hat{T}_{k,n+1}(t) + I^n_t(t + 8, \tau_2), \]

where \( T_{k,n+1} \) is the temperature index at day \( k \) of year \( n + 1 \), and \( \hat{T}_{k,n+1} \) is the temperature forecast for forecast days \( k = t, t + 1, \ldots, t + 7 \) in the current year issued at day \( t \)

- Historical weather forecasts
  - Point forecasts for maximum and minimum temperatures for the next 7 days
  - Issued and updated every 3 hours
  - Available at the National Digital Forecast Database
  - Only available after 06/06/2004
Chicago seasonal HDD 2005/06

Index 10 no detrending
CME market price

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Chicago seasonal HDD 2005/06
Chicago seasonal HDD 2005/06

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Results – Deviation of theoretical and market prices

Mean squared deviation of the prices observed at CME from index modeling prices with different numbers of years used. We analyse index modeling without detrending (wd), and index modeling with linear detrending (ld).

<table>
<thead>
<tr>
<th>$n$</th>
<th>MSE(wd)</th>
<th>MSE(ld)</th>
<th>$n$</th>
<th>MSE(wd)</th>
<th>MSE(ld)</th>
<th>$n$</th>
<th>MSE(wd)</th>
<th>MSE(ld)</th>
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<tr>
<td>1</td>
<td>6091.9</td>
<td>—</td>
<td>11</td>
<td>1212.8</td>
<td>2245.8</td>
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<td>—</td>
<td>12</td>
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<td>3</td>
<td>3084.8</td>
<td>25606.8</td>
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<td>1106.5</td>
<td>2372.5</td>
<td>23</td>
<td>1432.5</td>
<td>1579.3</td>
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<td>4</td>
<td>2140.5</td>
<td>15880.2</td>
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<td>1121.7</td>
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<td>24</td>
<td>1505.2</td>
<td>1966.9</td>
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<td>5</td>
<td>1235.8</td>
<td>11568.2</td>
<td>15</td>
<td>1190.9</td>
<td>1543.7</td>
<td>25</td>
<td>1540.3</td>
<td>2079.8</td>
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<tr>
<td>6</td>
<td>1152.4</td>
<td>7481.1</td>
<td>16</td>
<td>1264.3</td>
<td>1563.8</td>
<td>26</td>
<td>1683.1</td>
<td>2308.1</td>
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<td>7</td>
<td>1330.4</td>
<td>7383.5</td>
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<td>1281.1</td>
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<td>2040.5</td>
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<tr>
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<td>1589.4</td>
<td>5867.7</td>
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<td>30</td>
<td><strong>2425.9</strong></td>
<td>3160.1</td>
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</table>

With a mean value of the market prices of 1131.21 and a mean absolute error of 20.37, we calculate an average error of approximately 1.80% for the parsimonious index modeling pricing using 10 years of data with no detrending.
Results – Cross-correlations of theoretical and market prices

Cross-correlation of differentiated CME prices with pre-whitened differentiated index modeling prices using 10 years without detrending. *, **, and *** represent significance on a 10%, 5%, and 1% level, respectively.

<table>
<thead>
<tr>
<th>Lag</th>
<th>Correlation</th>
<th>Lag</th>
<th>Correlation</th>
<th>Lag</th>
<th>Correlation</th>
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<td>0.0088</td>
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<td>1</td>
<td>0.1676***</td>
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<td>-0.0120</td>
<td>21</td>
<td>-0.0281</td>
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<tr>
<td>2</td>
<td>0.0829***</td>
<td>12</td>
<td>-0.0366**</td>
<td>22</td>
<td>0.0179</td>
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<tr>
<td>3</td>
<td>0.0712***</td>
<td>13</td>
<td>0.0034</td>
<td>23</td>
<td>0.0699***</td>
</tr>
<tr>
<td>4</td>
<td>0.0271**</td>
<td>14</td>
<td>0.0123</td>
<td>24</td>
<td>0.0595***</td>
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<tr>
<td>5</td>
<td>0.0028</td>
<td>15</td>
<td>0.0132</td>
<td>25</td>
<td>0.0196</td>
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<tr>
<td>6</td>
<td>0.0290**</td>
<td>16</td>
<td>-0.0014</td>
<td>26</td>
<td>0.0053</td>
</tr>
<tr>
<td>7</td>
<td>-0.0005</td>
<td>17</td>
<td>0.0151</td>
<td>27</td>
<td>0.0292</td>
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<tr>
<td>8</td>
<td>0.0138</td>
<td>18</td>
<td>0.0212</td>
<td>28</td>
<td>0.0037</td>
</tr>
<tr>
<td>9</td>
<td>-0.0213</td>
<td>19</td>
<td>0.0026</td>
<td>29</td>
<td>-0.0177</td>
</tr>
</tbody>
</table>
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Trading strategies – General setting

- Use only monthly contracts
- Take the average of all CME prices between 10 and 20 days prior the measurement period
- Each month, allocate a fixed wealth to certain contracts
  - Usually hold until expiration
  - Square a contract if the loss exceeds 100%
- For each month, the return on margin is calculated as

\[ \text{ROM} = \frac{\text{final settlement price} - \text{initial market price}}{\text{initial margin} \times \text{initial market price}} \]
Trading strategies – Choice of contracts
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Chicago seasonal HDD 2005/06

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Trading strategies – Choice of contracts

Sell all HDD contracts with \( p_M < p^* \) and buy all CDD contracts with \( p_M > 1 - p^* \). Note that the choice of \( p^* \) is crucial to the success of the trading strategy and yet it yields a classical trade-off between the expected return and the availability of the contracts. When making \( p^* \) too large, we buy contracts where our model expects only a small return (or even a negative return for contracts with \( p^* > .5 \)). By making \( p^* \) too small the total return of the strategy may be limited due to the lack of matching contracts.

- **Strategy 1:** \( p^* = 1 \): Buy all CDD contracts, sell all HDD contracts
- **Strategy 2:** \( p^* = .5 \): Buy all CDD contracts with \( M < I^{30} \), sell all HDD contracts with \( M > I^{30} \) (\( M = \) market price, \( I^{30} = \) Index Modeling price with detrending and 30 years of data used)
- **Strategy 3:** \( p^* = .44 \), calibrated to maximal profit for virtual contracts from 1980–2000
Trading strategies – Results

Overview of the different trading strategies for the period of June 2002–September 2006. The values in parentheses indicate the \( p \)-values of the one-sided Ledoit and Wolf (2008) test whether the corresponding Sharpe ratio is less or equal to the S&P 500 Futures Sharpe ratio. ** indicates significant excess returns on a 5% level.

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P 500 Futures</th>
<th>Strategy 1 ( p^* = 1 )</th>
<th>Strategy 2 ( p^* = .5 )</th>
<th>Strategy 3 ( p^* = .44 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Monthly Return</td>
<td>1.3%</td>
<td>54.1%**</td>
<td>70.2%**</td>
<td>102.7%**</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>28.0%</td>
<td>222.7%</td>
<td>288.1%</td>
<td>483.0%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>4.2%</td>
<td>24.3%</td>
<td>28.0%</td>
<td>21.3%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(10.3%)</td>
<td>(13.3%)</td>
<td>(14.3%)</td>
</tr>
<tr>
<td>Total Return</td>
<td>69.9%</td>
<td>2,652.7%</td>
<td>3,089.3%</td>
<td>3,593.1%</td>
</tr>
</tbody>
</table>
Conclusion

- Market prices can be replicated well with an index model using 10 years of data and applying no detrending
- Since weather data inhibits trends, the market prices are biased

**Market price of risk**

CDD prices are too low, HDD prices are too high

- Positive MPR for CDD, negative MPR for HDD?
- Data suggests MPR in the weather market is merely an academic issue
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June 01, 2010

21 / 21